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Model-Free Data-Driven Computing

Michael Ortiz

California Institute of Technology and

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25th Engineering Mechanics Symposium

Arnhem, the Netherlands

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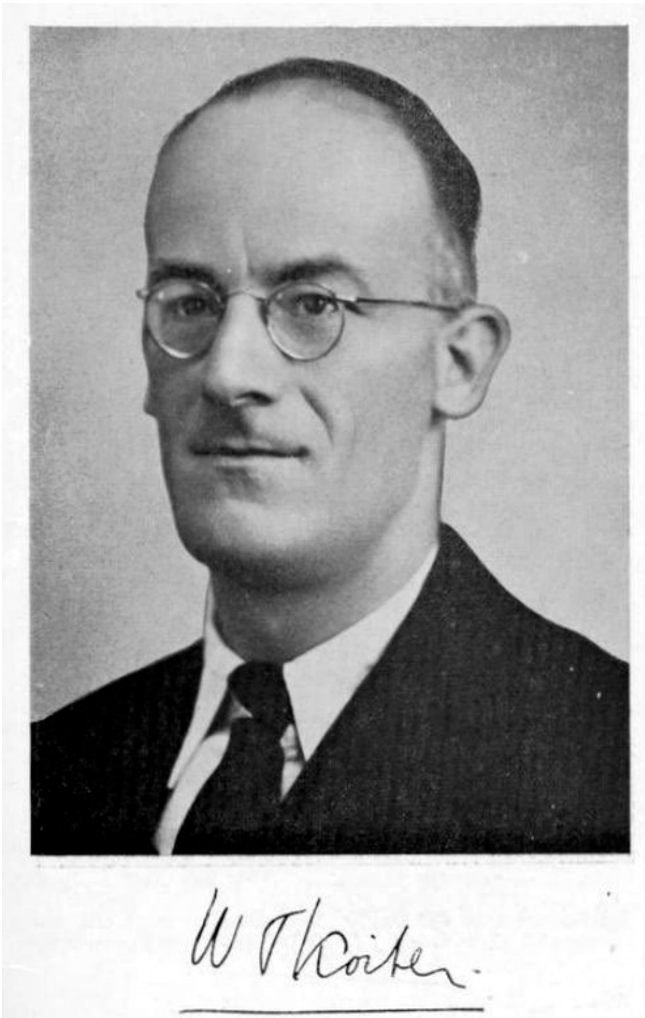
To Warner Tjardus Koiter, *in memoriam*



Amsterdam, June 16, 1914
Delft, September 2, 1997

- Dedicated, *grato animo*, to the memory of *Warner T. Koiter*, eximious engineer and educator, for his pioneering contributions to linear and non-linear thin shell theory, plasticity, elasticity and applied mathematics

A Translation of
THE STABILITY OF ELASTIC EQUILIBRIUM

By

WARNER TJARDUS KOITER

Sponsored by
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Sunnyvale, Calif.

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Why Data-Science now?

Data Science, Big Data, AI…
What's in it for us?

http://olap.com/forget-big-data-lets-talk-about-all-data/

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Why Data-Science now?

- New emerging paradigm: *Data-Driven Science*
- Fundamental premise: Unprecedented *abundance of material data*, be it *experimental* or from *multiscale analysis*

The new data-rich world of EM…

- Material *data is currently plentiful* due to dramatic advances in *experimental science* (DIC, EBSD, microscopy, tomography…) and *multiscale analysis* (DFT → MD → DDD → SM → Hom)

3D tomographic reconstruction
of particles in battery electrode

3D DIC-measured
internal-strain full-field
compressed PDMS sample

John Lambros, UIUC,
https://lambros.ae.illinois.edu/moviesimages/

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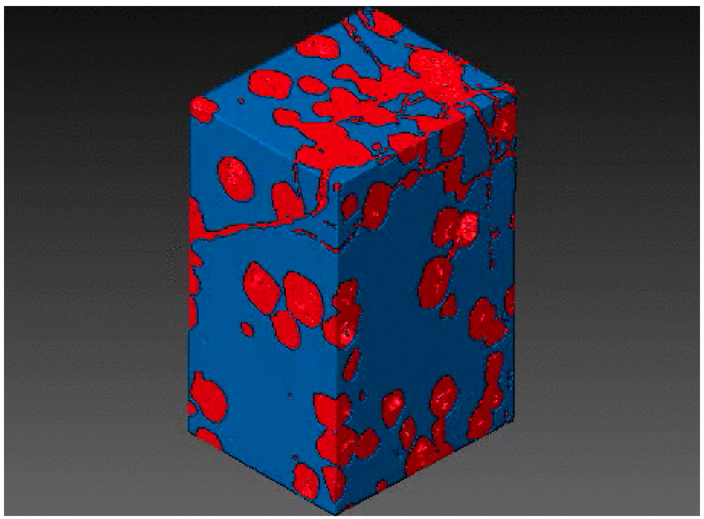
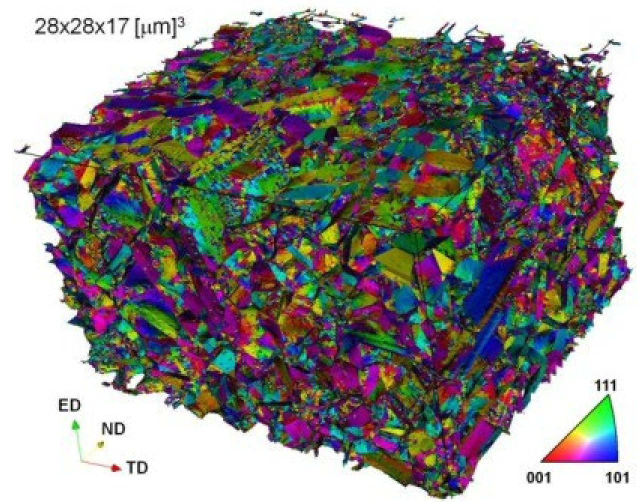


Two-phase μCT analysis of Ti2AlC/Al composite[1]



3D EBSD microstructure in Cu-0.17wt%Zr after ECAP[2]

[1]Hanaor *etal*, *Mater Sci Eng A*, **672** (2019) 247.
[2]Khorashadizadeh, *Adv Eng Mater*, **13** (2011) 237.

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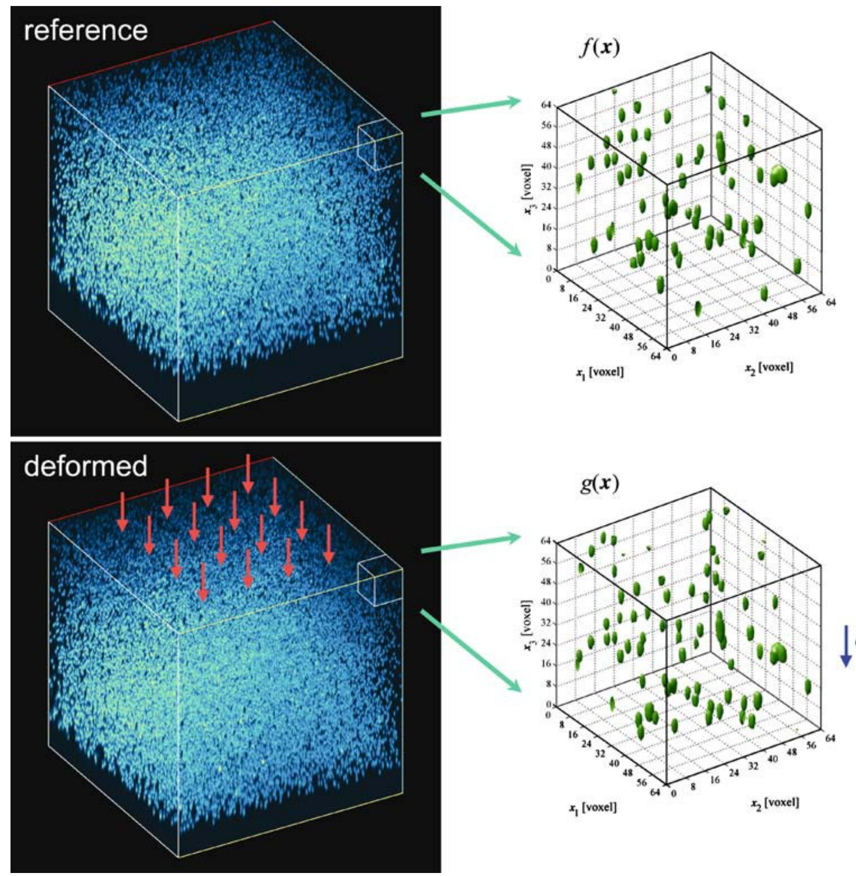
Digital Volume Correlation (DVC): Two confocal volume images of an agarose gel with randomly dispersed fluorescent particles before and after mechanical loading. The full displacement vector field is measured using 3D volume correlation methods.



C. Franck, S. Hong, S.A. Maskarinec, D.A. Tirrell and G. Ravichandran, *Experimental Mechanics* (2007) **47**:427–438.

The new data-rich world of EM…

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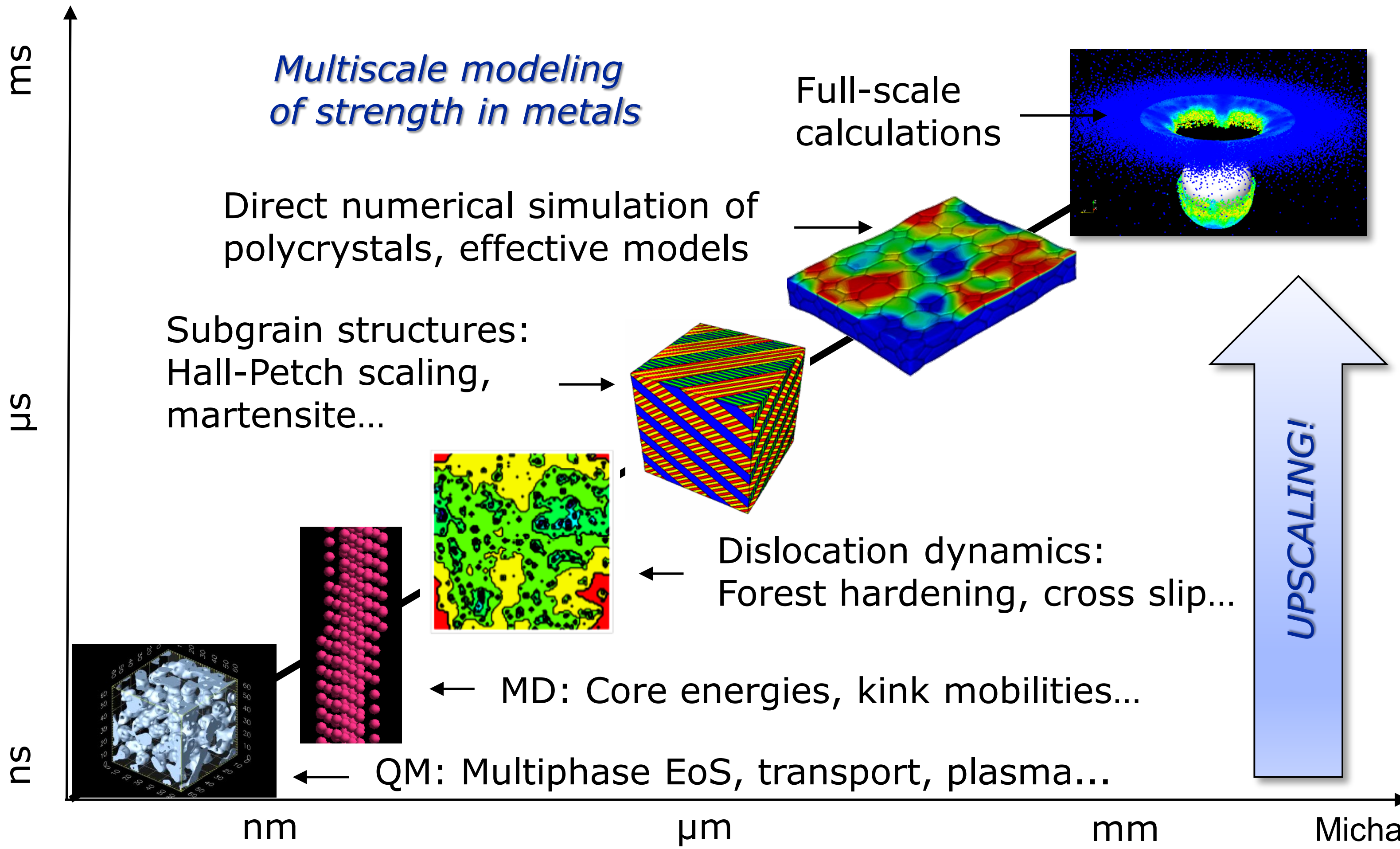

The new data-rich world of EM…

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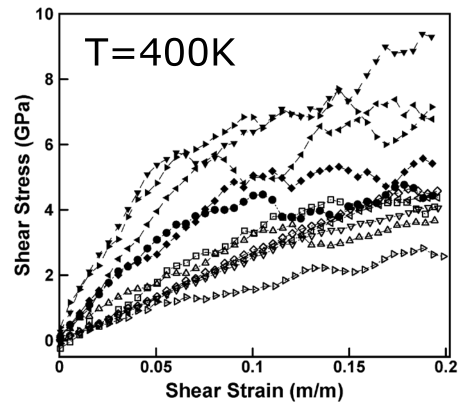
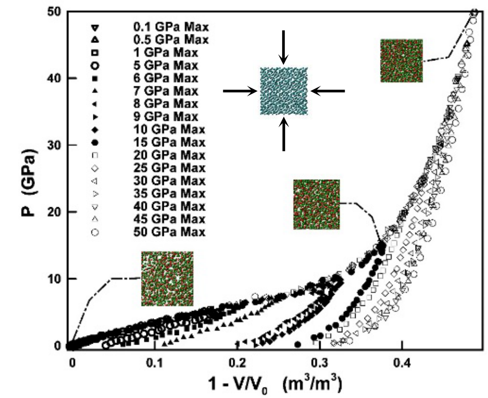
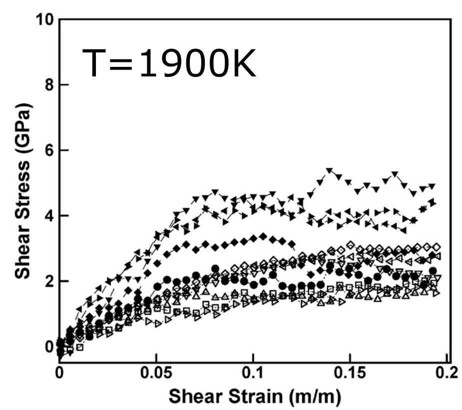
Amorphous SiO_2 glass: LAMMPS MD calculations of amorphous silica glass under *pressure-shear* loading over a range of *temperatures* and *strain rates*. RVEs are quenched from the melt, then analyzed using the BKS potential with Ewald summation.

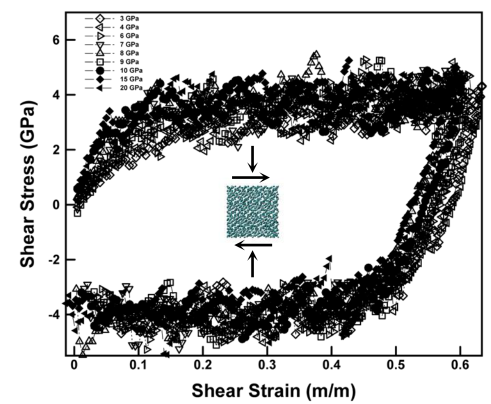
Schill, W., Heyden, S., Conti, S. & MO, *JMPS*, **113** (2018) 105-125.

Schill, W., Mendez, J.P., Stainier, L. & MO, *JMPS*, **140** (2020) 103940.









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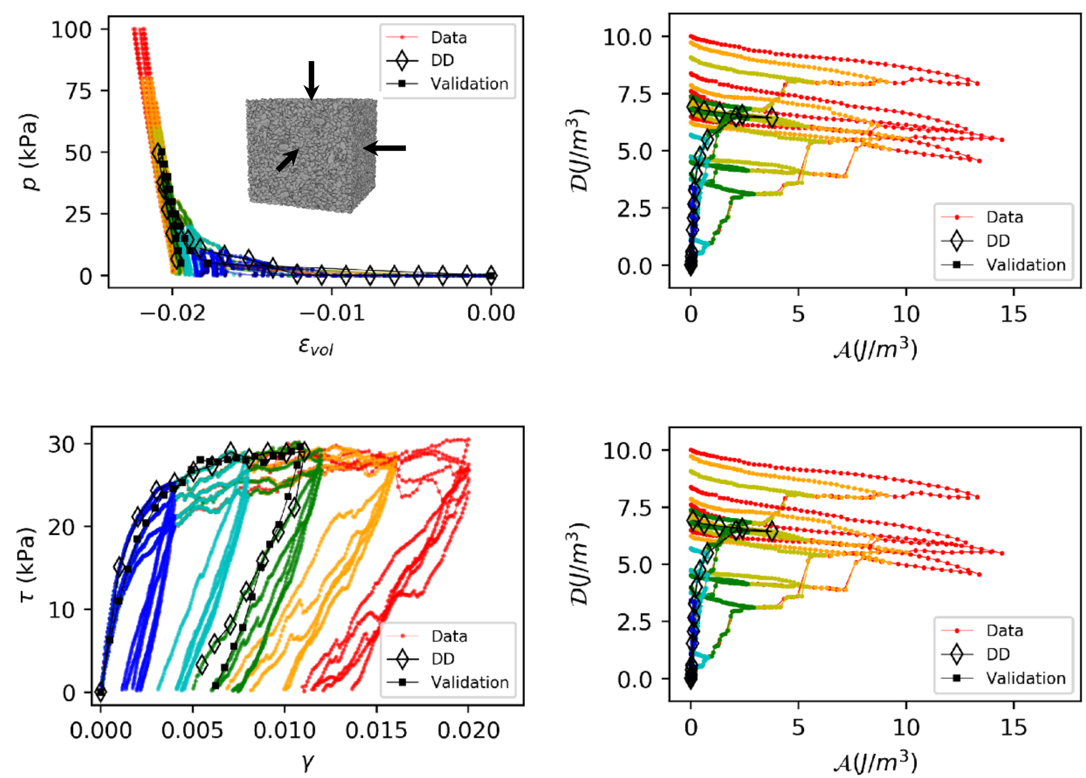
- Material *data is currently plentiful* due to dramatic advances in *experimental science* (DIC, EBSD, microscopy, tomography…) and *multiscale analysis* (DFT → MD → DDD → SM → Hom)

Granular matls. (dry sand): Level-Set Discrete Element Method (LS-DEM) simulation of granular material samples. 3D irregular *rigid particles* interact through *frictional contact*. Particle morphology described by level-set functions. Note calculation of *dissipation and free energy*.

Karapiperis, K., Harmon, J., And, E., Viggiani, G. & Andrade, J.E., *JMPS*, **144** (2020) 104103.

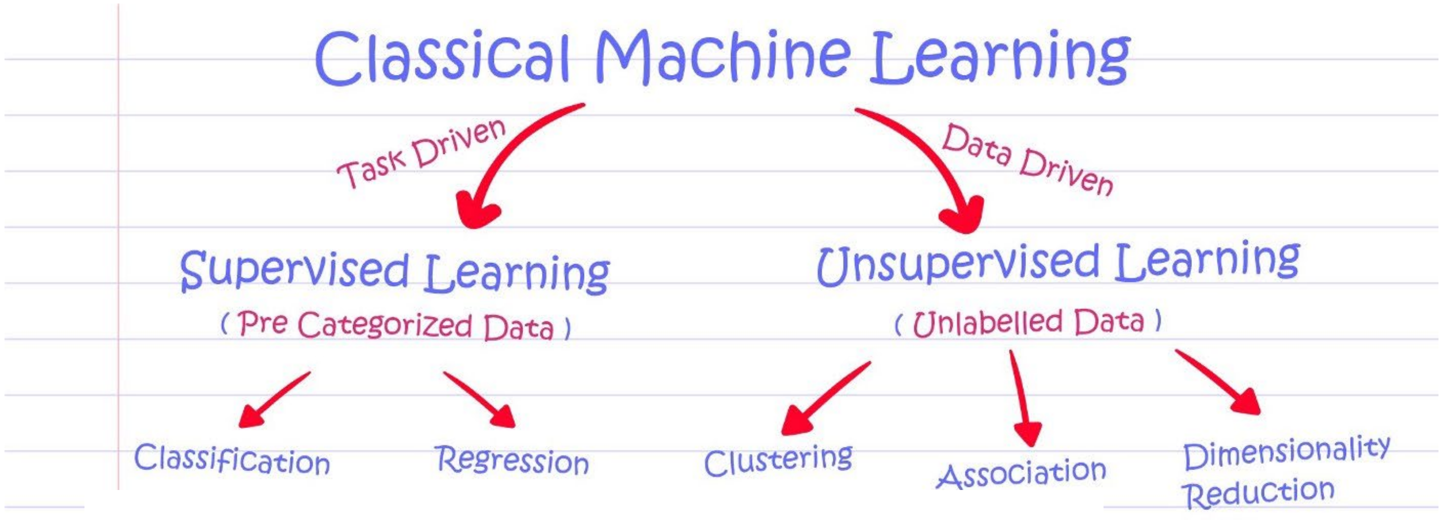
Karapiperis, K., Stainier, L., Ortiz, M. & Andrade, J.E., *JMPS*, **147** (2021) 104239.



Why Data-Science now?

- New emerging paradigm: *Data-Driven Science*
- Underlying premise: Unprecedented *abundance of material data*, be it *experimental* or from *multiscale analysis*
- Challenge: Forge a *closer connection between data and predictions!*
- Fundamental dilemma: *To fit or not to fit* (that is the question)

Model-Based Data-Driven computing:	Data	$\rightarrow$	Model	$\rightarrow$	Prediction
Model-Free Data-Driven computing:	Data		$\longrightarrow$		Prediction



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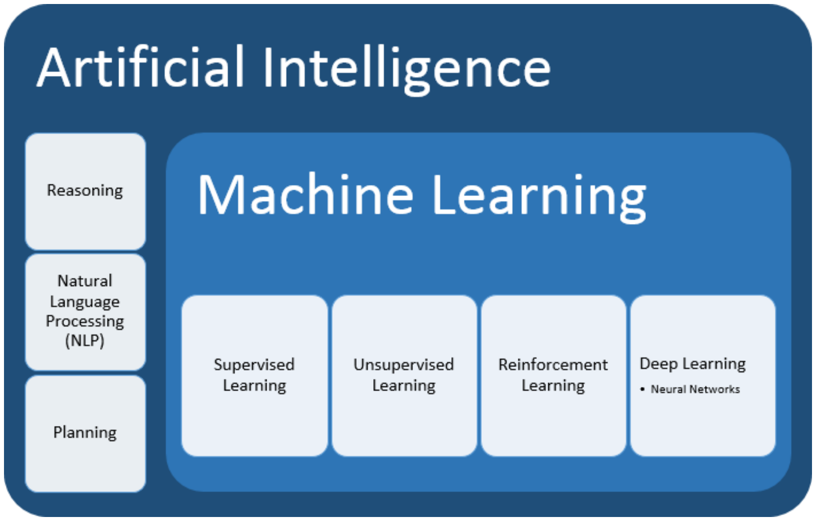
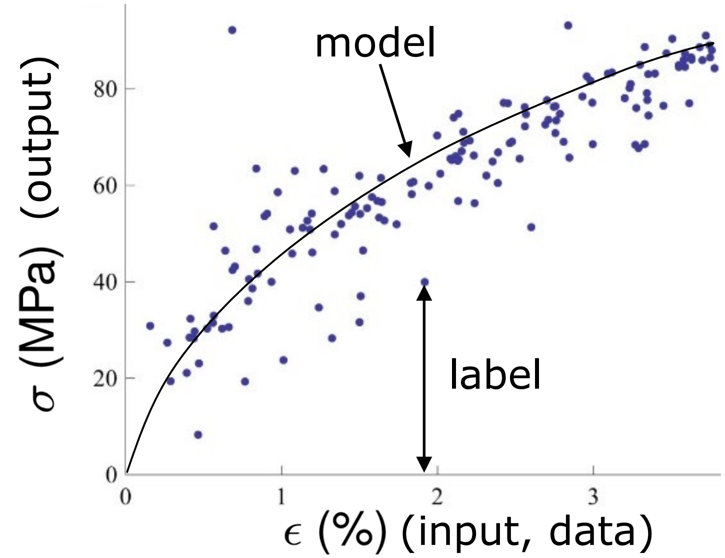
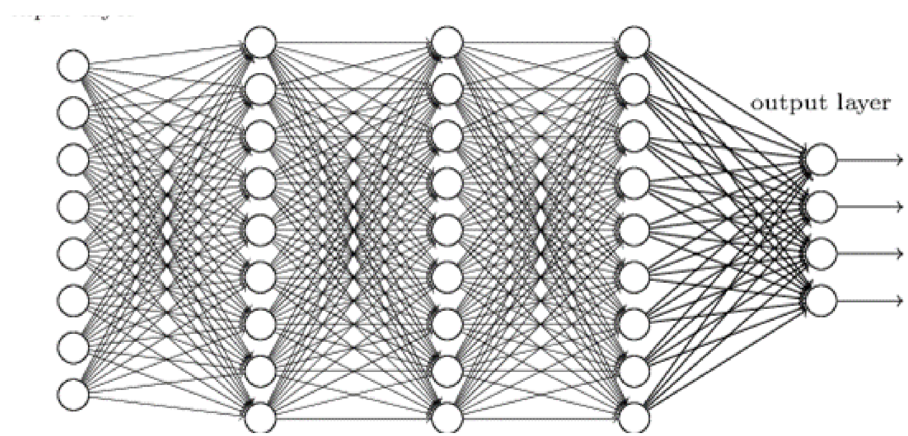
To fit or not to fit, that is the question





Supervised learning: Find (e.g., by ***regression***) a function (e.g., ***deep Neural Network***) from data containing both inputs and outputs (labels).

J. Hurwitz & D. Kirsch, *Machine Learning*, John Wiley & Sons, 2018.



Deep Neural Network representation, regression

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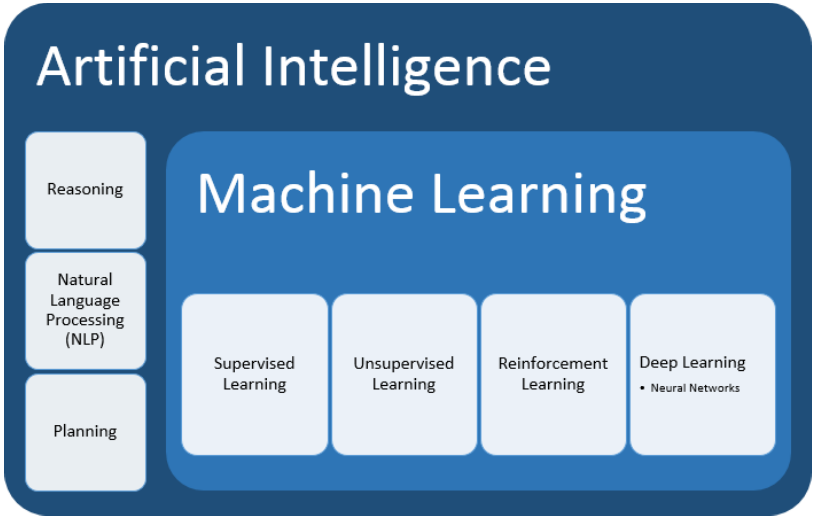
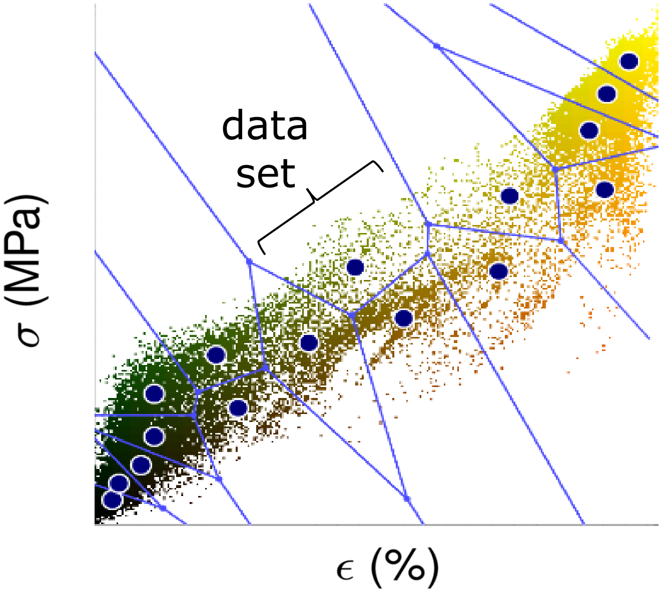
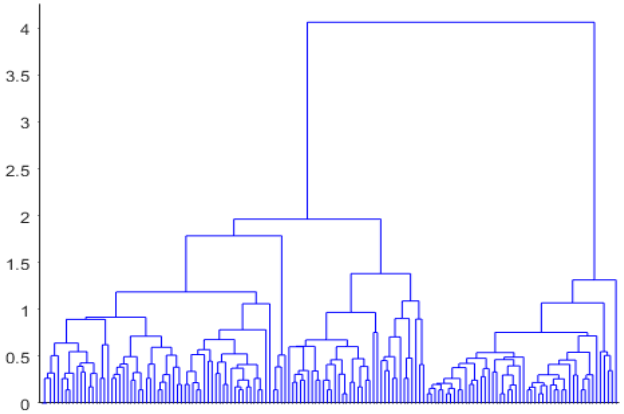
To fit or not to fit, that is the question





Unsupervised learning: Find structure in *unlabeled data* sets (grouping, clustering, density, *data mining*), make predictions directly from *data structures*.

J. Hurwitz & D. Kirsch, *Machine Learning*, John Wiley & Sons, 2018.



Hierarchical k-means representation, set based

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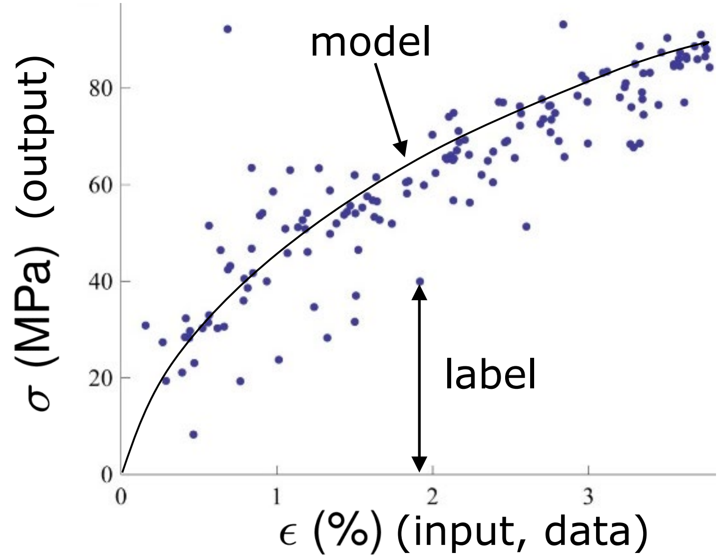
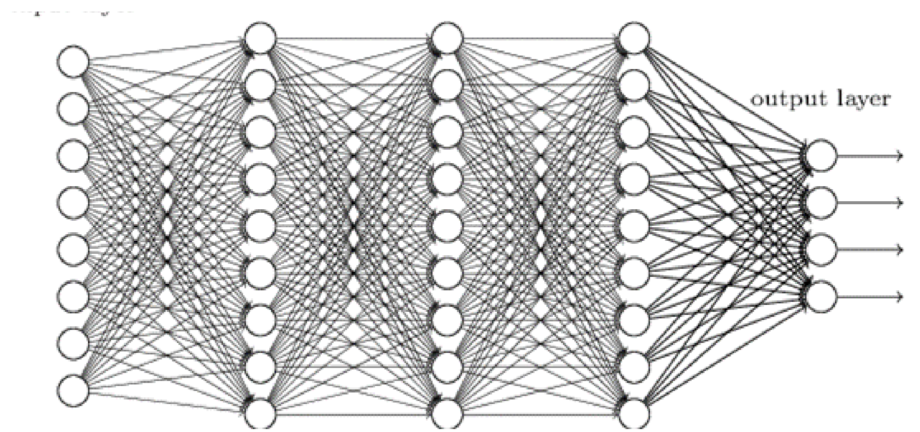
Supervised machine learning, pros and cons

- *Advantages:*
  - *Universal approximation theorem for C^0 functions*
  - *Powerful for regression of data in high dimensions*
  - *Availability of versatile tools in the public domain*
  - *Easy to integrate into FEM*

- *Disadvantages:*
  - *Loss of information with respect to data set*
  - *Modelling error, bias (e.g., choice of network topology)*
  - *Poor error estimation, control of convergence wrt data*
  - *Poor extrapolation properties*
  - *Poor physical interpretability*
  - *Enormous number of hidden parameters, costly training*
  - *Slower than classical analytical material laws*





Deep Neural Network representation, regression

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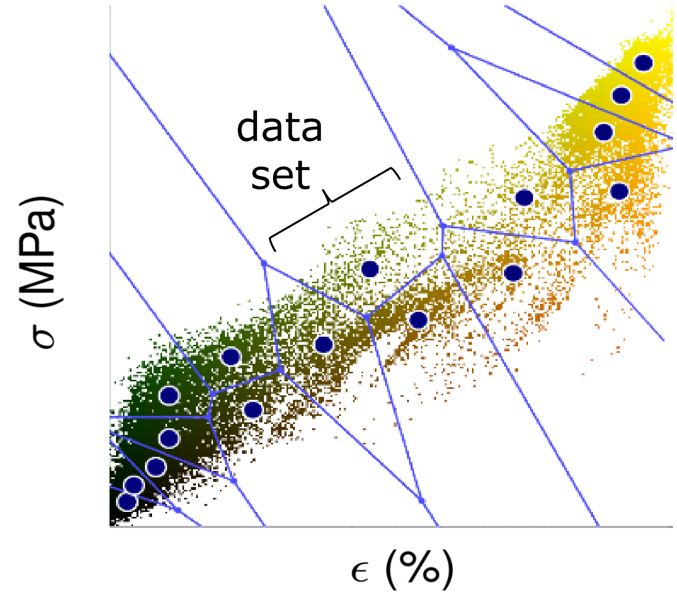
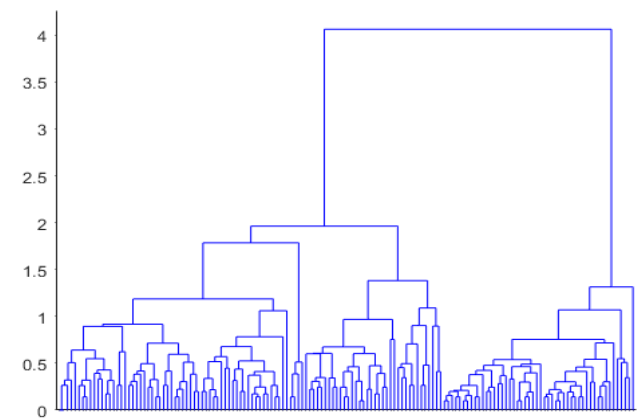
Unsupervised machine learning, pros and cons

- *Advantages:*
  - *Lossless: The data, all the data, nothing but the data*
  - *No modeling error, bias*
  - *No prior ansaetze, modeling*
  - *Powerful error estimates, convergence wrt to data*
  - *Standardization of solvers, material data repositories*
  - *Concurrent data generation, multiscale analysis, RVEs*
  - *Faster than conventional solvers (e.g., Newton-Raphson)*

- *Disadvantages:*
  - *Specialized solvers for FEM (software interfaces, scripting)*
  - *Fast data searching algorithms, efficient data structures*

How?





Hierarchical k-means representation, set based

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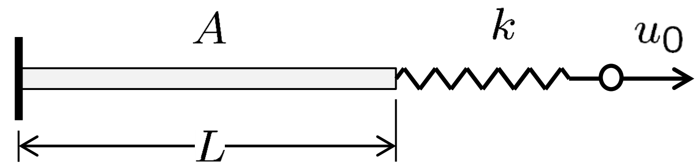
The MFDD paradigm (unsupervised ML)

The model-free Data-Driven paradigm

- *Phase space*, $Z = \{(\epsilon, \sigma)\}$.
- Note (ϵ, σ) work-conjugate
- Dimension of Z is even
- Compatibility: $\epsilon = u/L$
- Equilibrium: $\sigma A = k(u_0 - u)$
- Eliminate u: $\sigma A = k(u_0 - \epsilon L)$



Definition (Constraint set)

The constraint set is the affine subspace of Z containing all admissible states (ϵ, σ) satisfying compatibility and equilibrium:

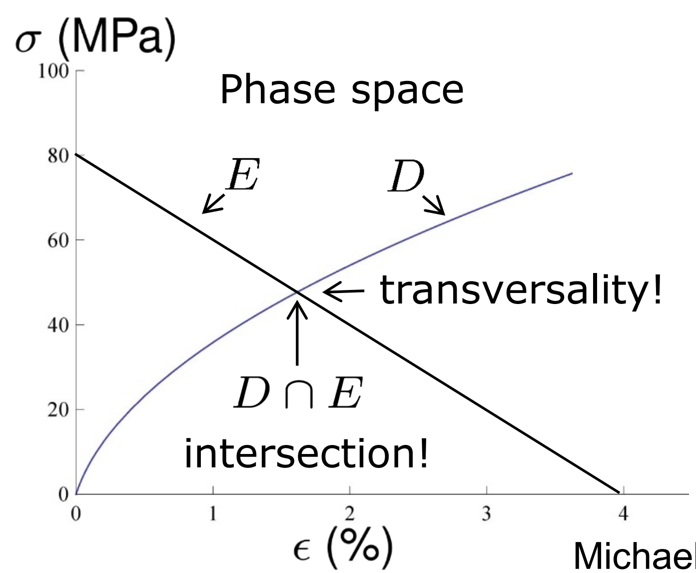
$$E = \{(\epsilon, \sigma) \,:\, \sigma A = k(u_0 - \epsilon L)\}$$

Definition (Material data set)

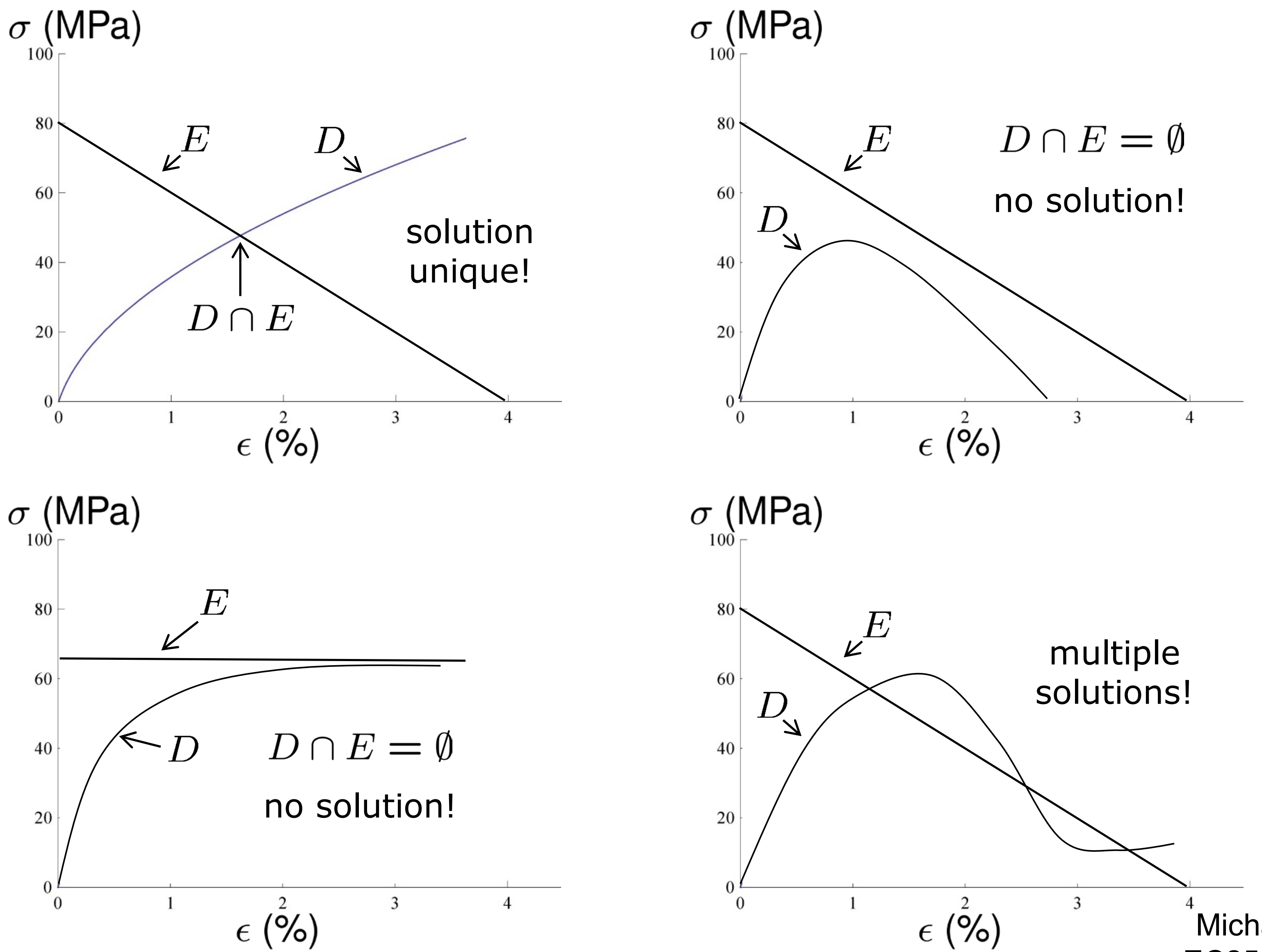
The material data set D is the subset of Z containing all the observed states (ϵ, σ).

Definition (Classical solution)

The classical solution is the intersection $D \cap E$, i. e., the set of all material states that are admissible.



The model-free Data-Driven paradigm



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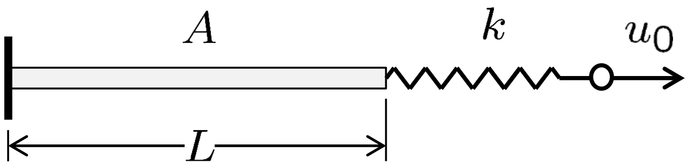
The model-free Data-Driven paradigm

- Suppose that $D =$ point set
- Then, $D \cap E = \emptyset$
- No classical solutions! Must extend the concept of solution, classical approach is too rigid



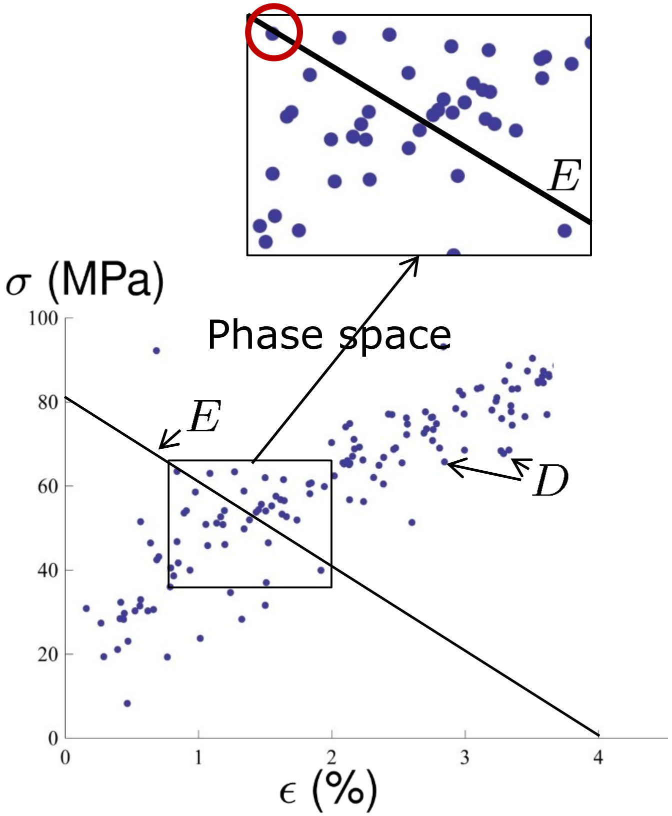
Definition (Data-Driven solution)

An admissible state $z \in E$ is a Data-Driven solution if it minimizes the distance to the material data set D,

$$\mathrm{dist}(z, D) \to \min!, \quad z \in E$$

- Recall: $\mathrm{dist}^2(z, D) = \min_{y \in D} \|y - z\|^2$
- Data-Driven problem:

$$\min_{z \in E} \min_{y \in D} \|y - z\|^2 = \min_{y \in D} \min_{z \in E} \|y - z\|^2$$



- Find material state $y \in D$ and admissible state $z \in E$ closest to each other.

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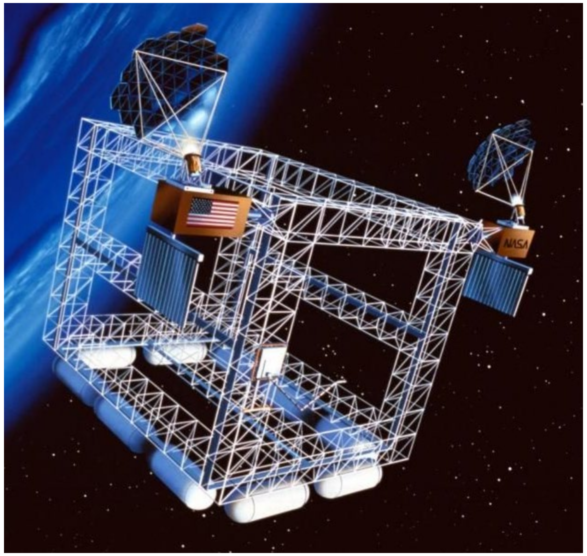
The model-free Data-Driven paradigm



Definition (Data-Driven Problem)

Given phase space $Z = \mathbb{R}^N \times \mathbb{R}^N$,

i) $D = \{\text{material data}\} \subset Z$,

ii) $E = \{\text{field equations}\} \subset Z$,

Find: $\text{argmin}\{\|y - z\|^2 \,:\, y \in D,\ z \in E\}$

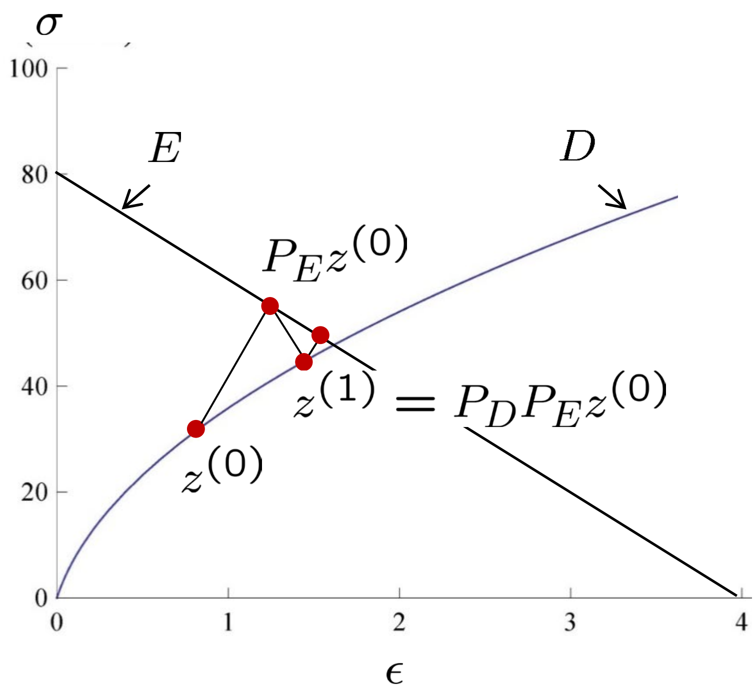
- Discussion:
  - Phase space Z determined by field equations
  - Fundamental data (model-independent) $=$ Points in phase space
  - No material modeling, no loss of information, no biasing of the data
  - DD problem generalizes and subsumes classical field-theoretical problems
- Outlook:
  - Extensions to infinite dimensions? (e.g., linear elasticity)
  - Extensions to geometrically-nonlinear problems? (e.g., finite elasticity)
  - Well-posedness of Data-Driven problems? Convergence with respect to data?
  - Solvers? Computational performance? Scaling?

Need solvers!

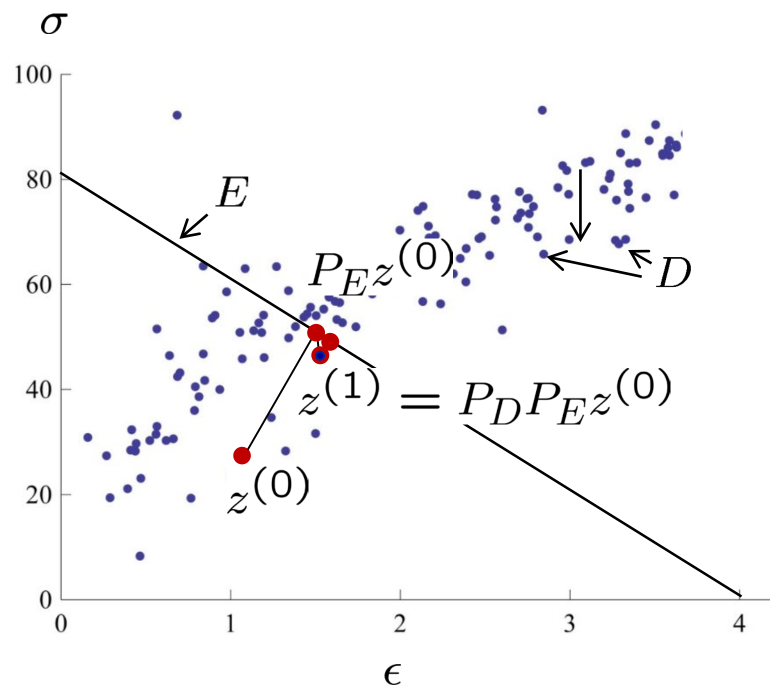
Solvers: Fixed-point iteration

- Find: $\text{argmin}\{\text{dist}(z, D),\ z \in E\}$
- Solver: $z^{(k)} = P_D \circ P_E z^{(k-1)}$
  - $P_D :=$ closest-point projection onto D.
  - $P_E :=$ closest-point projection onto E.

- *Implementation?*
- *Convergence?*

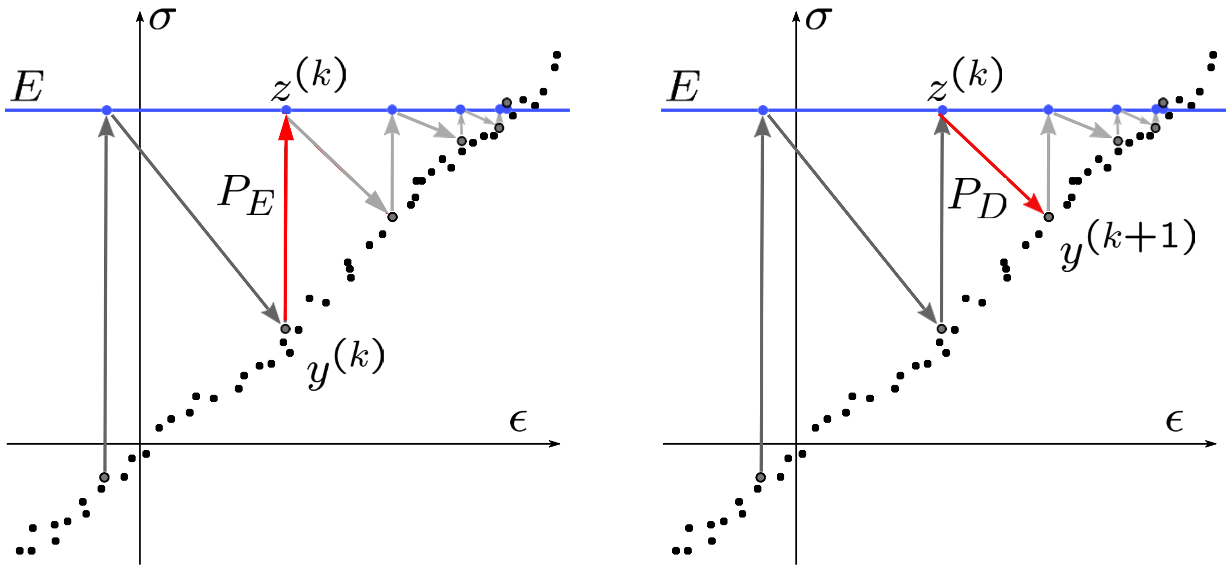


Fixed-point iteration,
manifold data set D



Fixed-point iteration,
point data set D

Solvers: Fixed-point iteration

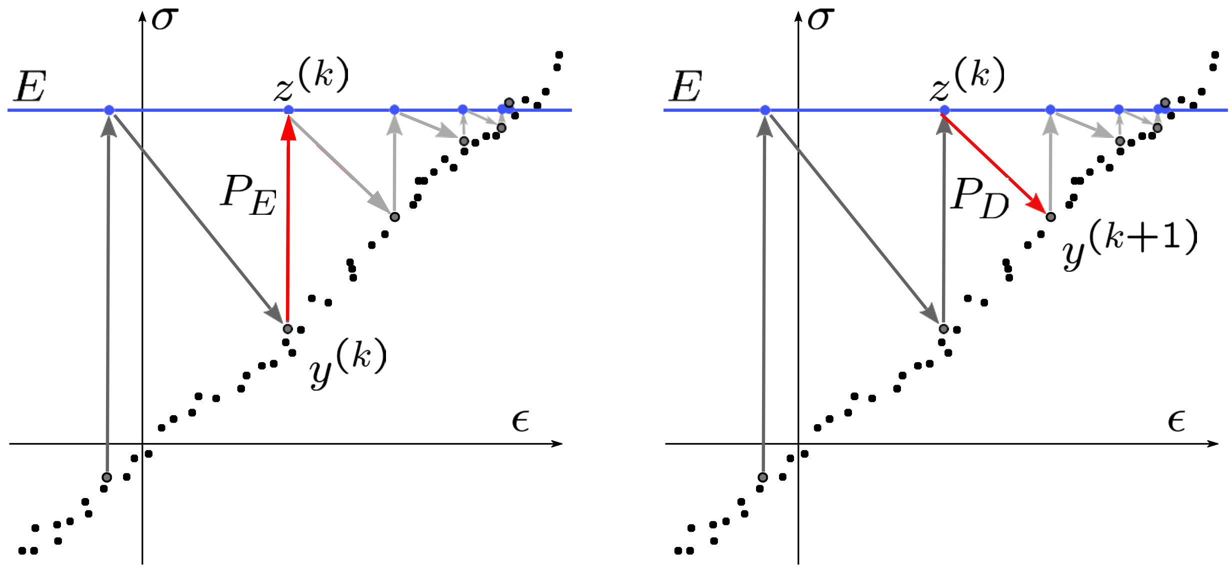


- Projection to E (inner minimization at fixed (ϵ', σ')): i) Enforce compatibility directly by writing $\epsilon = Bu$; ii) Enforce equilibrium through a Lagrange multiplier v,

$$\delta\Big(\|(Bu - \epsilon', \sigma - \sigma')\|^2 - (B^T W \sigma - f) \cdot v \Big) = 0$$

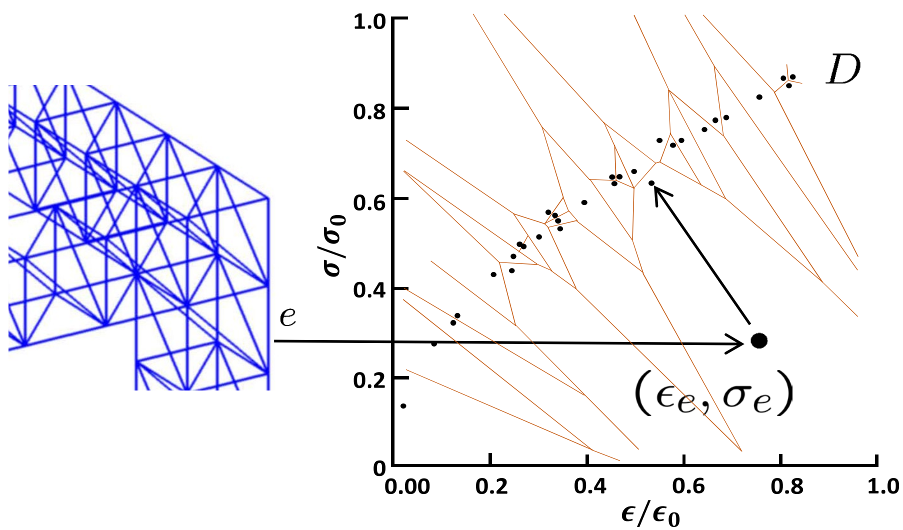
- Euler-Lagrange equations: $(B^T \mathbb{C} W B) u = B^T \mathbb{C} \epsilon', \quad (B^T \mathbb{C} W B) v = f - B^T \sigma'$.
- State update: $\quad \epsilon = Bu; \quad \sigma = \sigma' + \mathbb{C} B v$.
- Two standard linear problems! (regardless of material behavior).
- DD leads to (material-independent) standardization of solvers.

Solvers: Fixed-point iteration

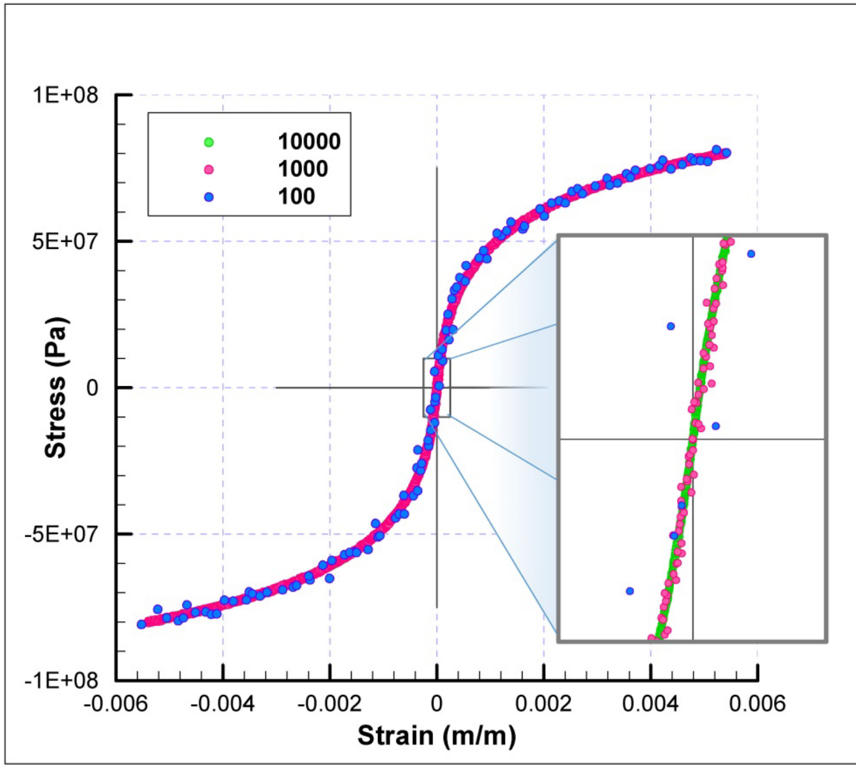


- Outer minimization: Projection onto (closest point in the) material data set D
- Fast searching algorithm
- Requires data structures
- 'Learning' structure of D
- Set-oriented (lossless) ML!

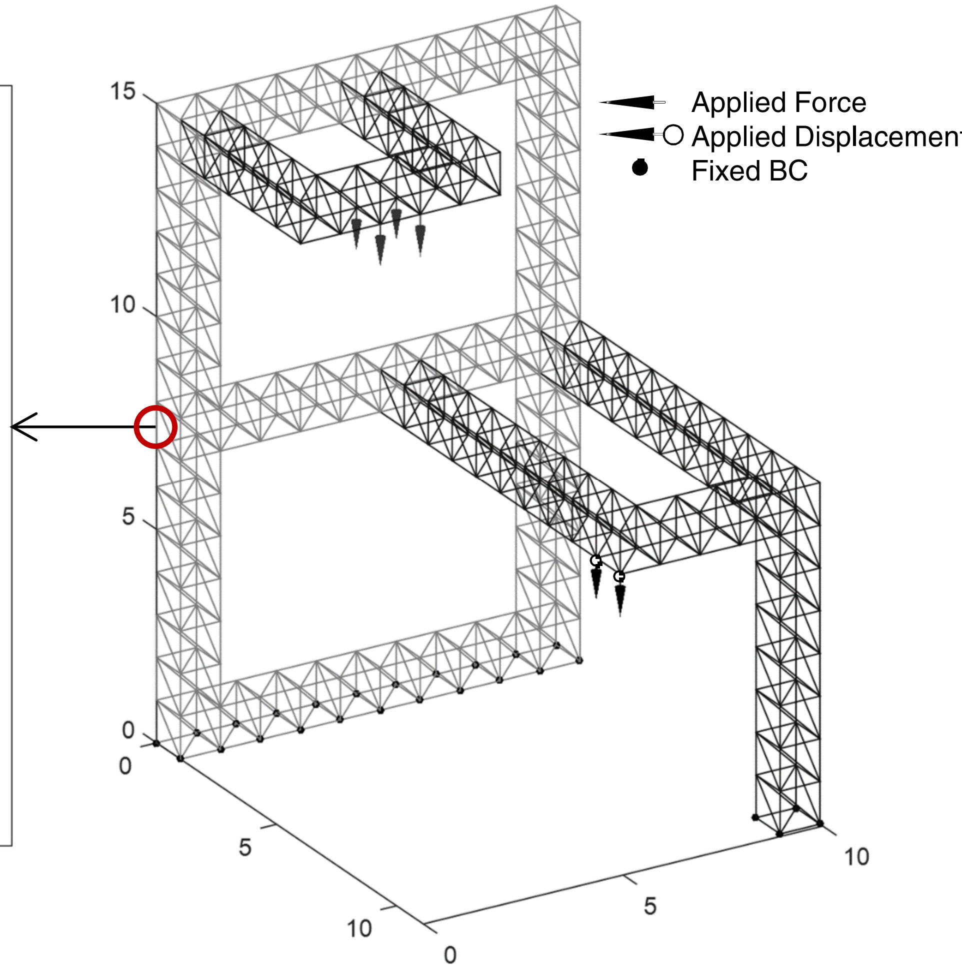
Unsupervised machine learning!



Convergence with respect to the data



Sequence of
uniformly converging data sets
(increasing number of points, decreasing scatter)



3D truss structure

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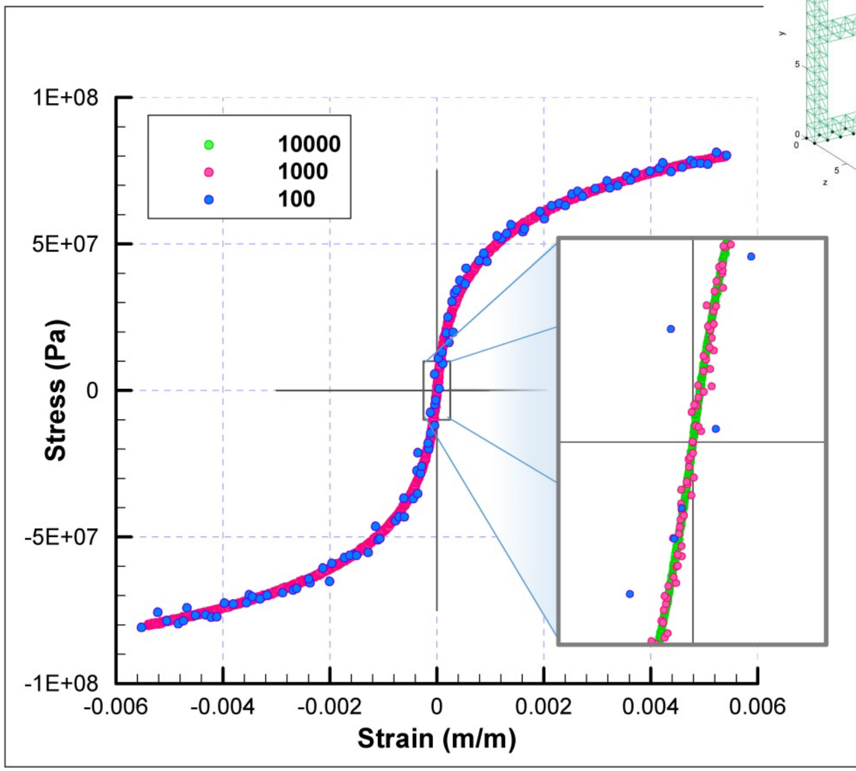
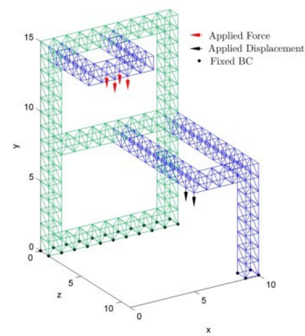
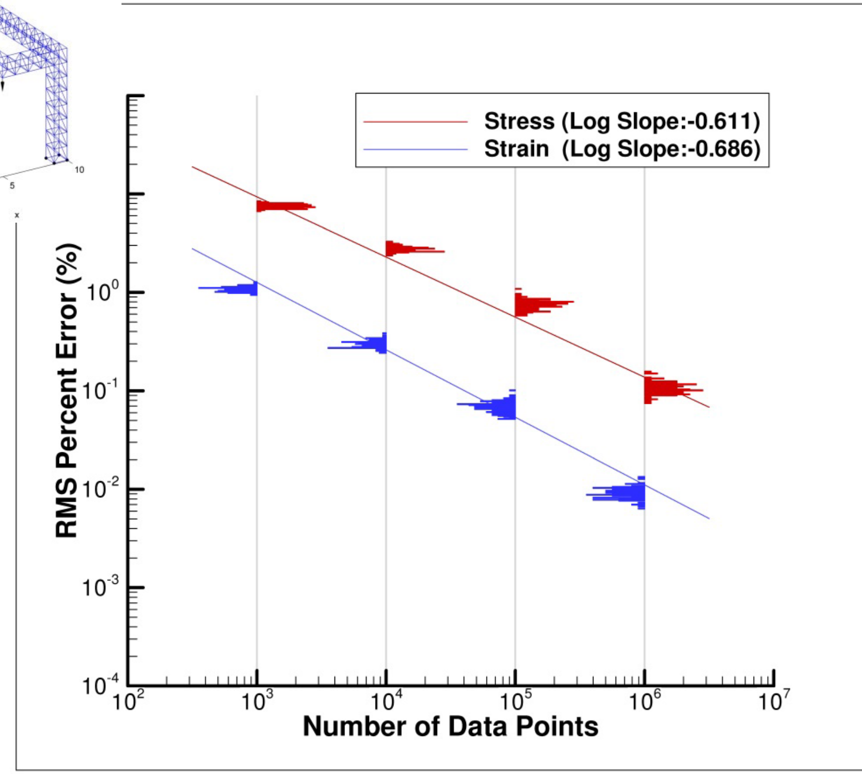
T. Kirchdoerfer and M. Ortiz, *CMAME*, **304** (2016) 81-101.

Convergence with respect to the data





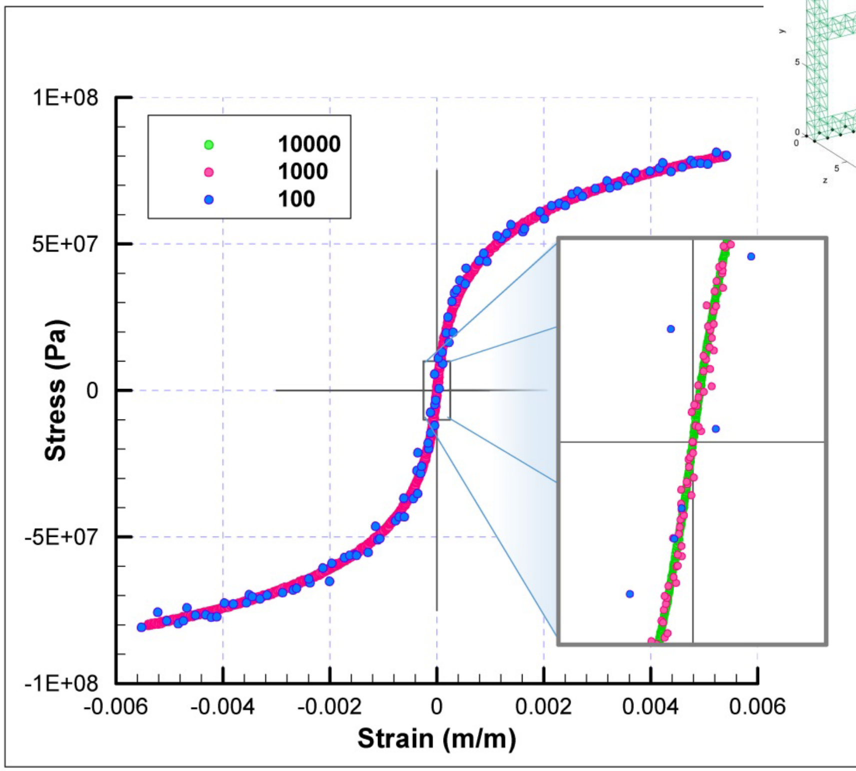


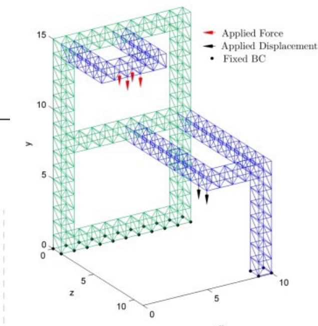
Sequence of
uniformly converging data sets
(increasing number of points,
decreasing scatter)

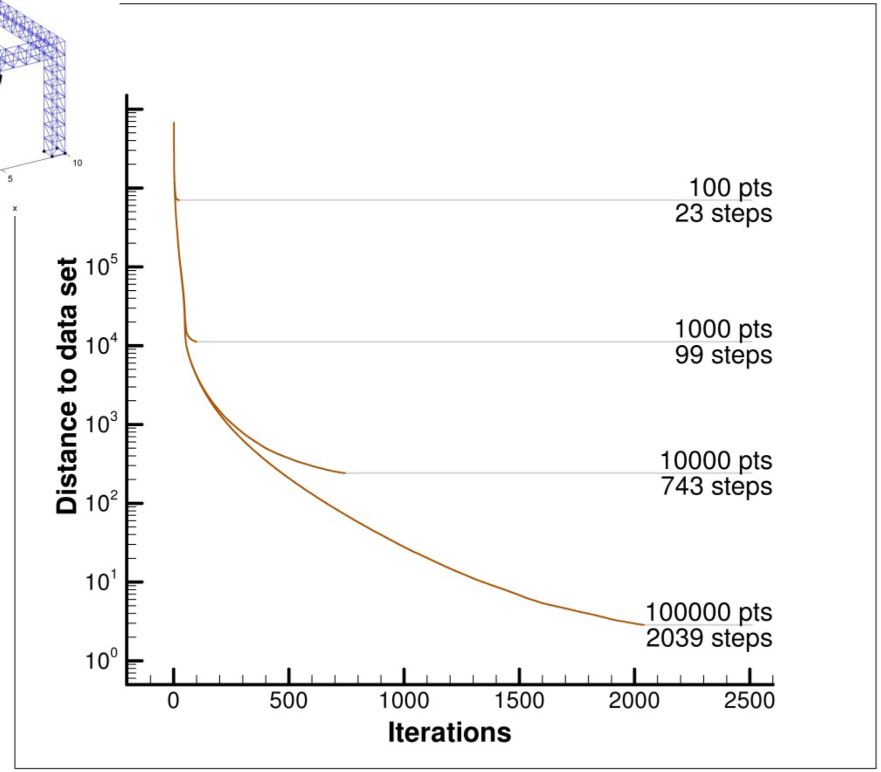
Convergence with respect to material data set towards
solution of limiting problem
(nonlinear elasticity)

T. Kirchdoerfer and M. Ortiz, *CMAME*, **304** (2016) 81-101.

Solvers: Fixed-point iteration







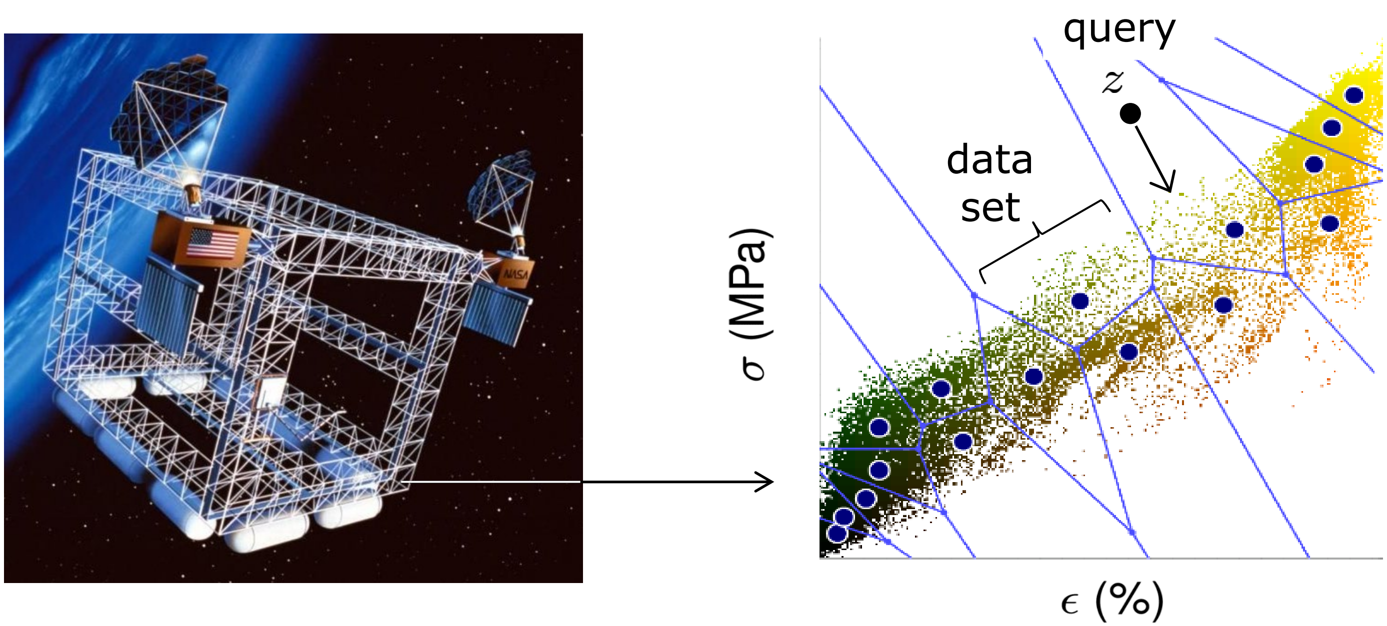
Sequence of *uniformly converging data sets* (increasing number of points, decreasing scatter)

Convergence of fixed-point solver: Each iteration requires two back-substitutions for standard linear systems and one material data search/member

T. Kirchdoerfer and M. Ortiz, *CMAME*, **304** (2016) 81-101.

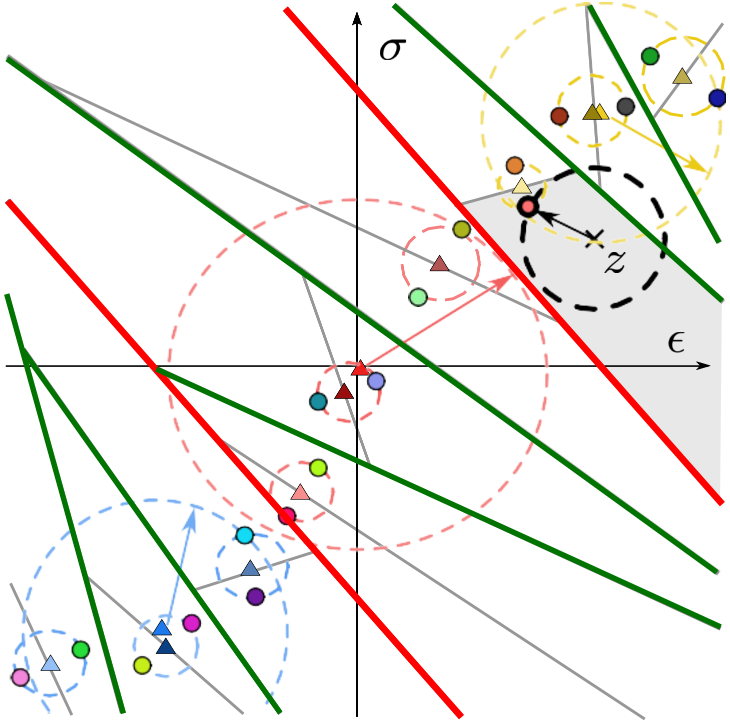
Dealing with big data (unsupervised ML)

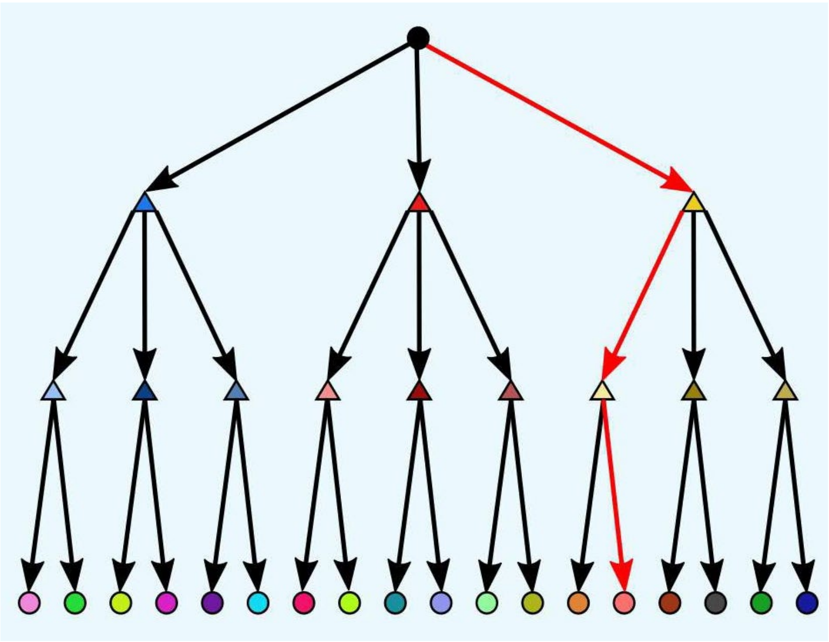
Big data sets – Unsupervised learning



- Material data sets can be very large, cannot do naïve linear lookups!
- *k-means clustering* aims to partition *n* points into *k* clusters in which each point belongs to the cluster with the *nearest mean* (centroid).

Big material data sets – k-means





Nearest-neighbor tree traversal

- Material data sets can be very large, cannot do naïve linear lookups!
- *k-means clustering* aims to partition *n* points into *k* clusters in which each point belongs to the cluster with the *nearest mean* (centroid).
- *k*-means clustering can be applied *recursively* to define a *k-means tree*
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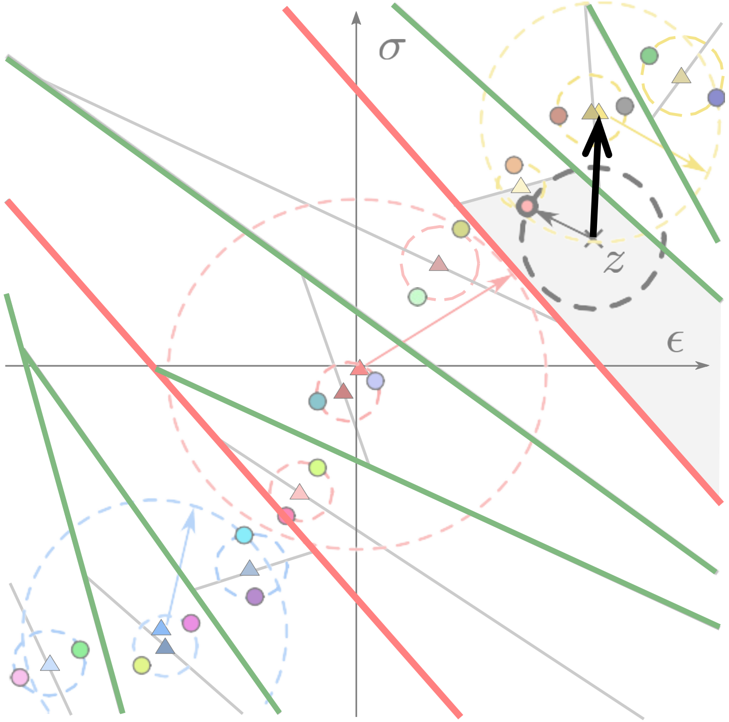
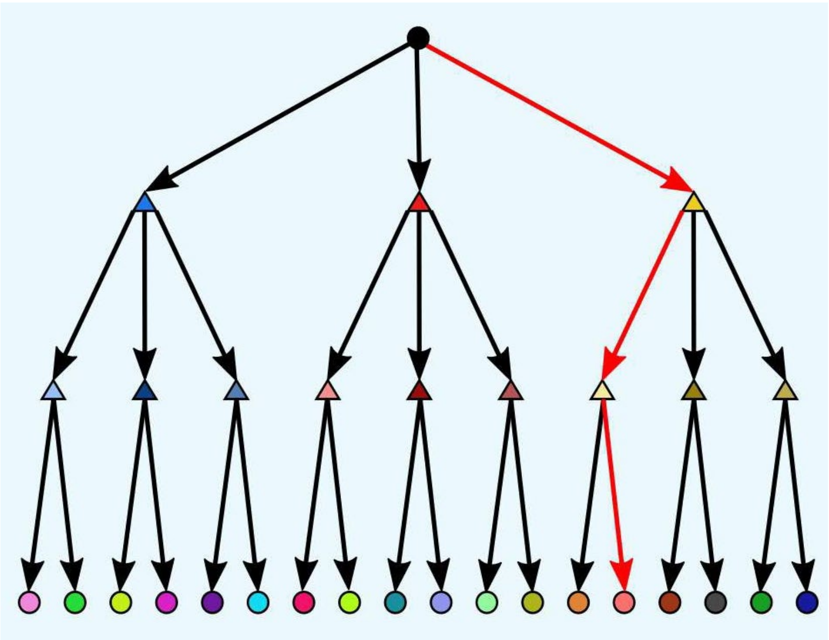
Big material data sets – k-means





Nearest-neighbor tree traversal

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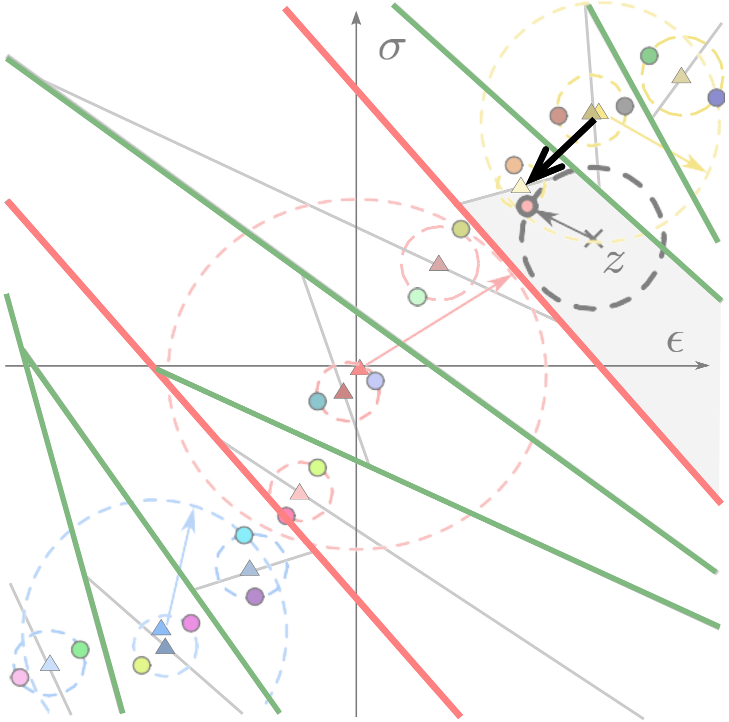

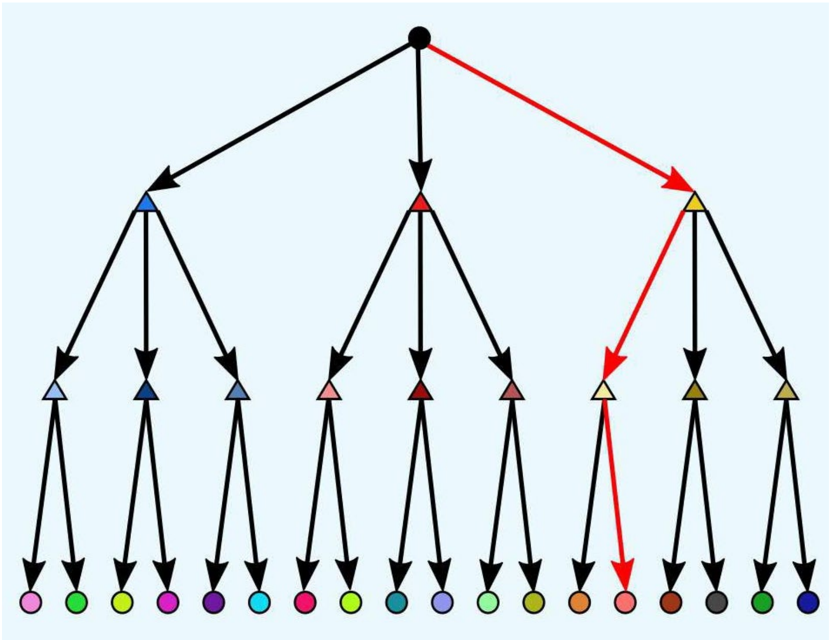

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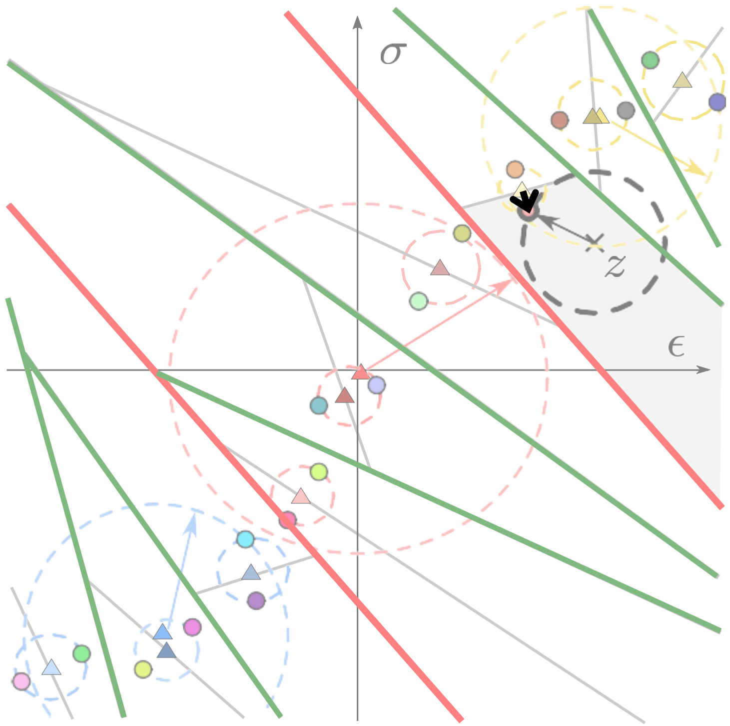

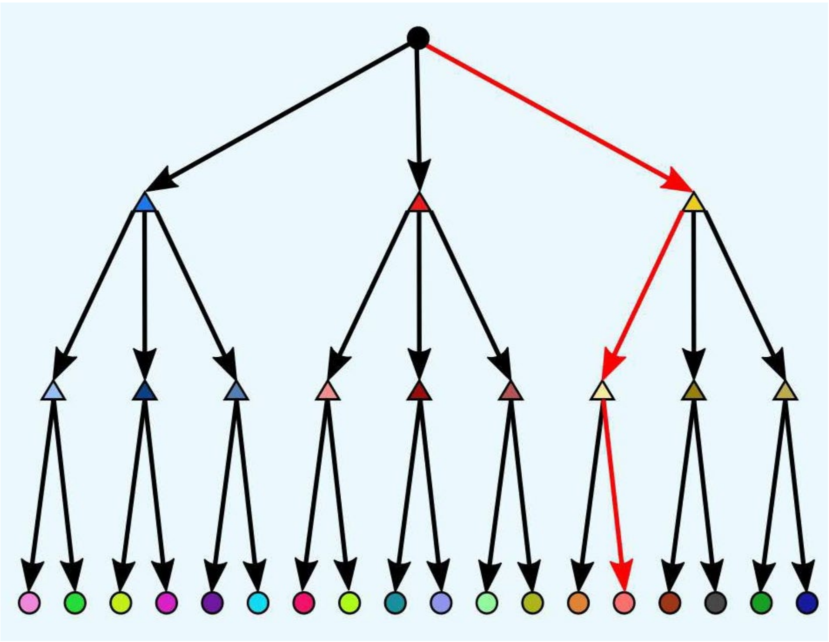

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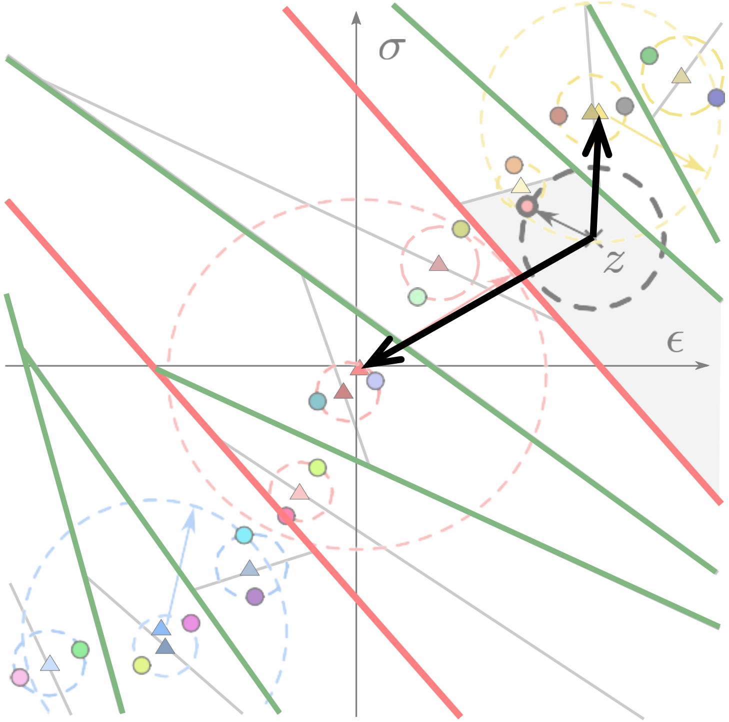

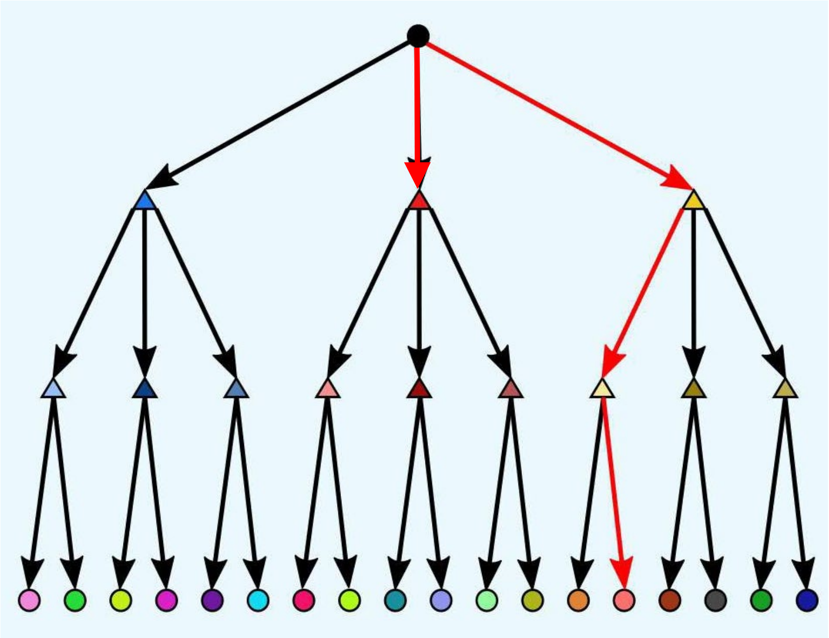

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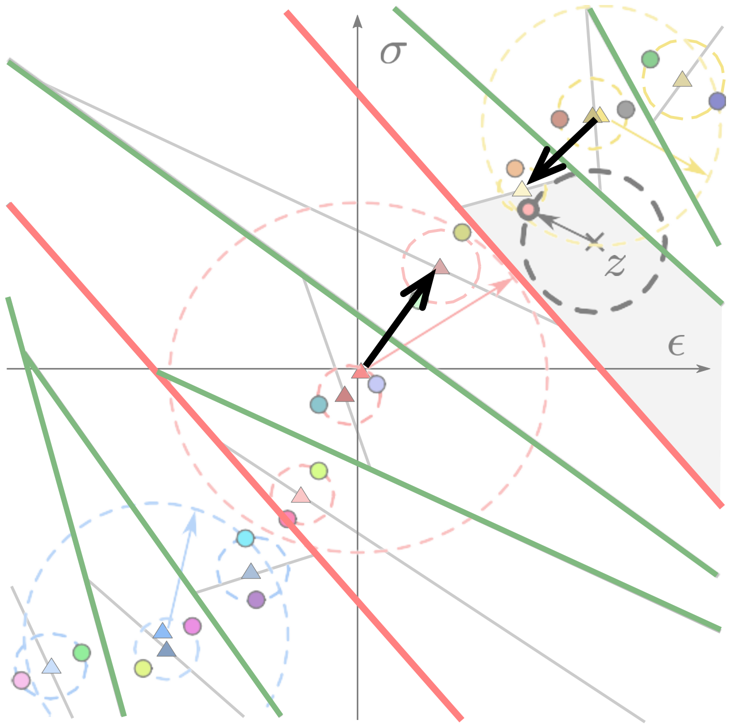

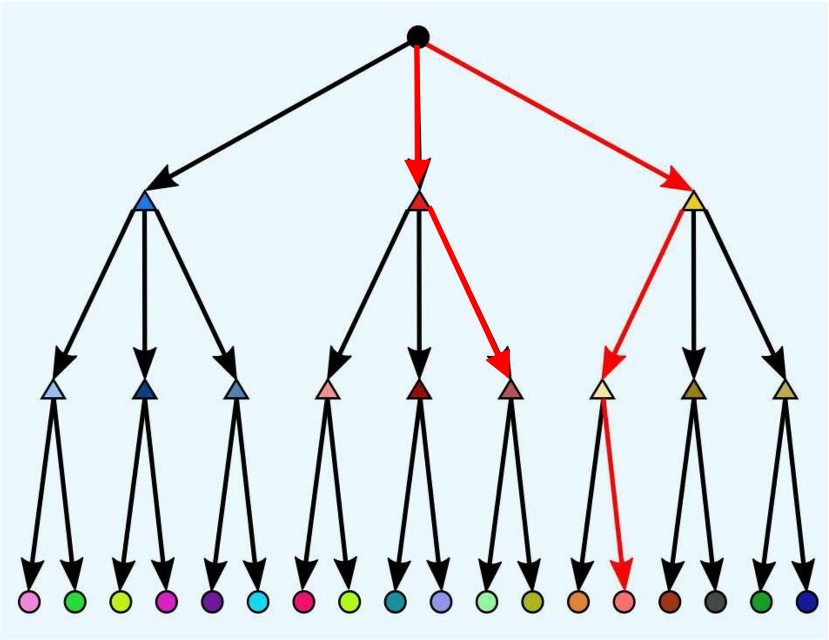

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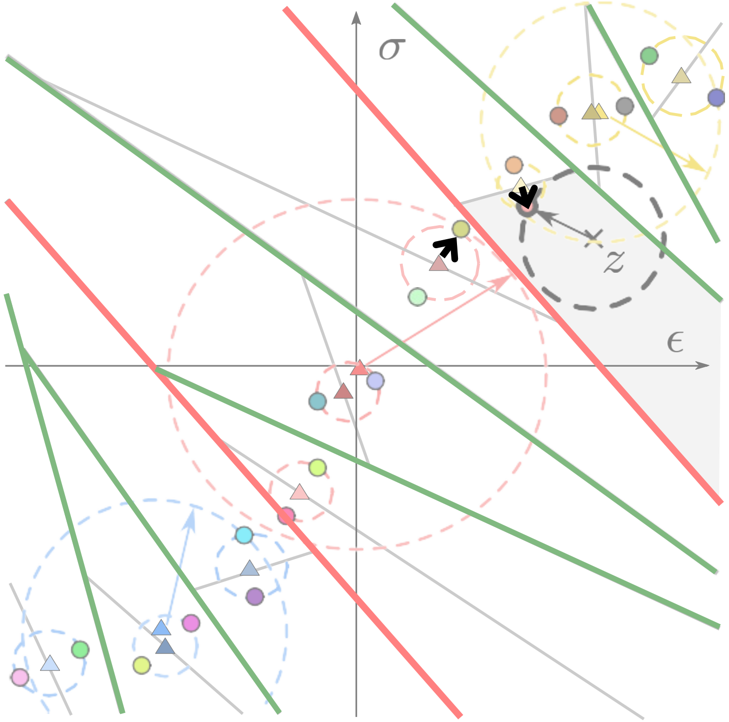

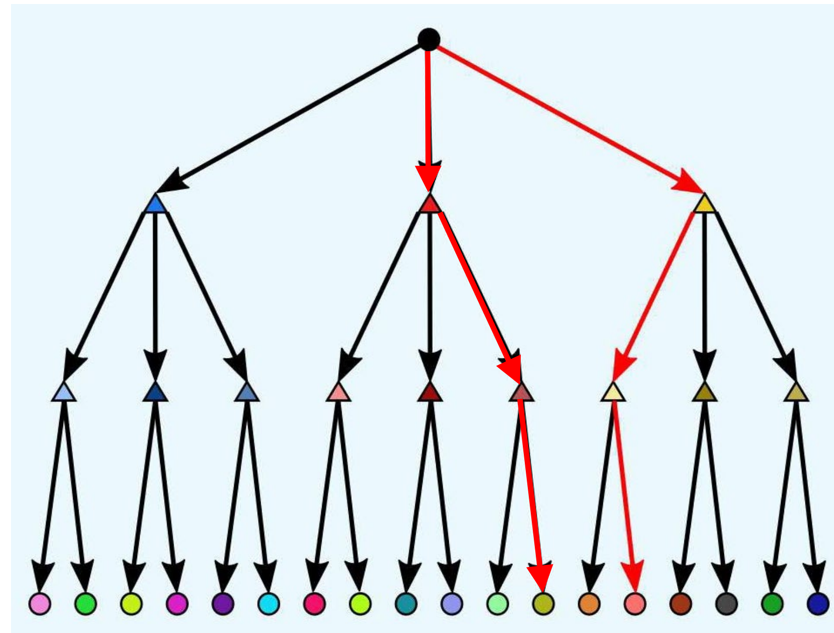

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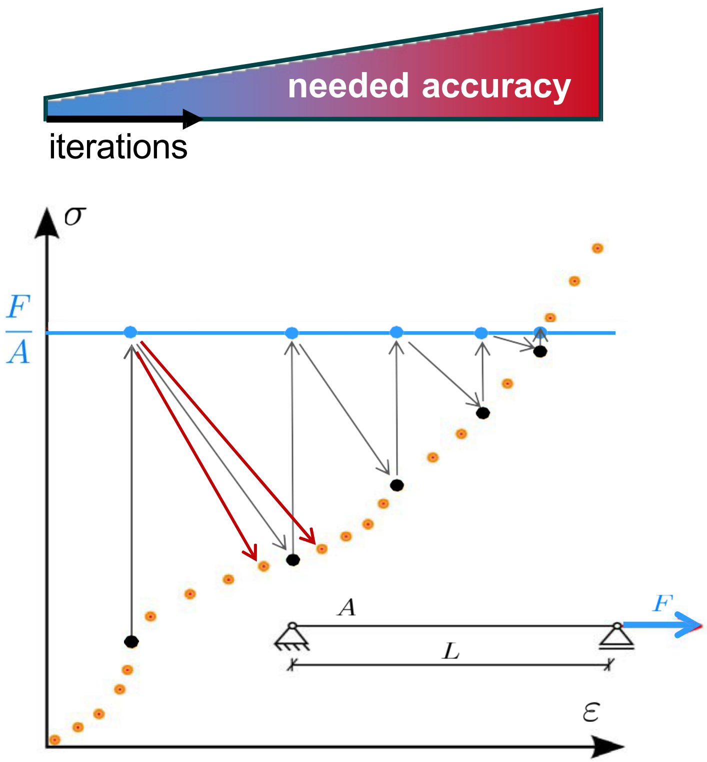
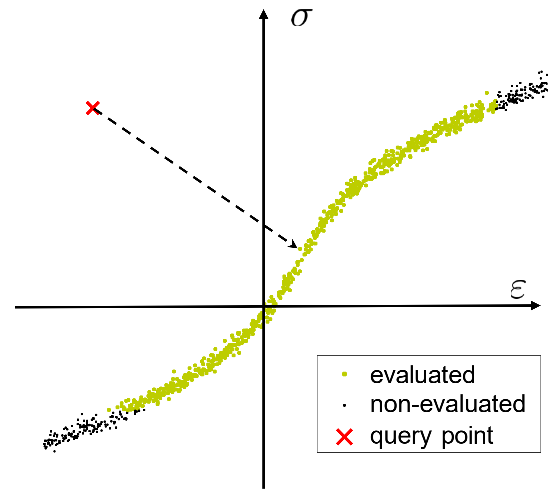
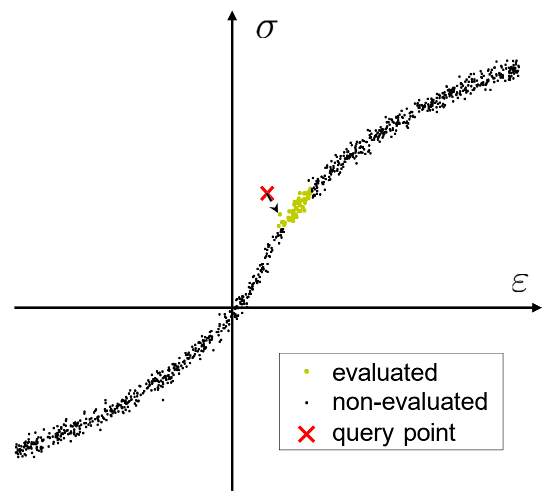




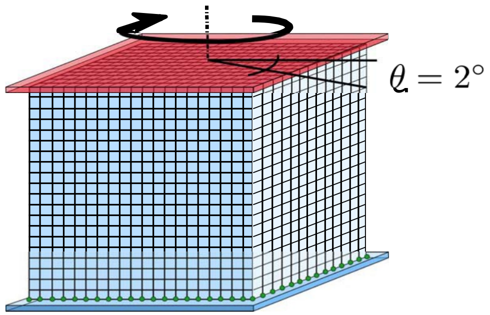
Initial iterations



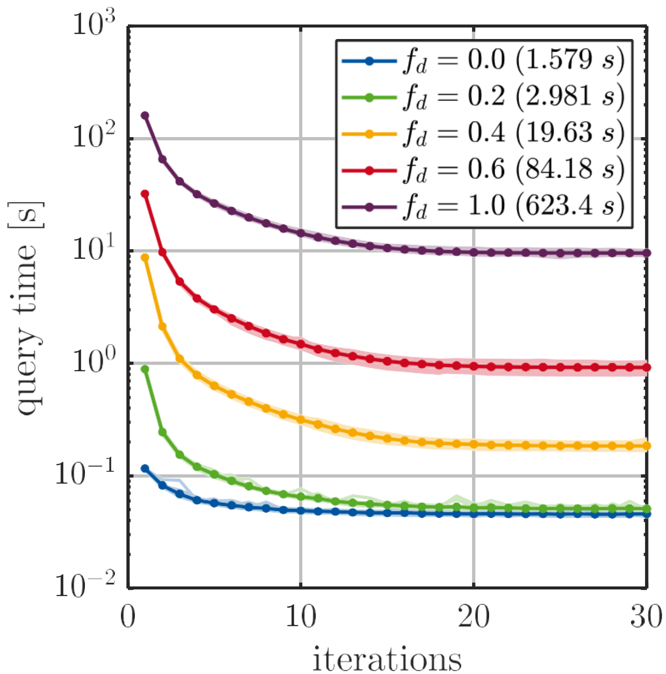
Final iterations

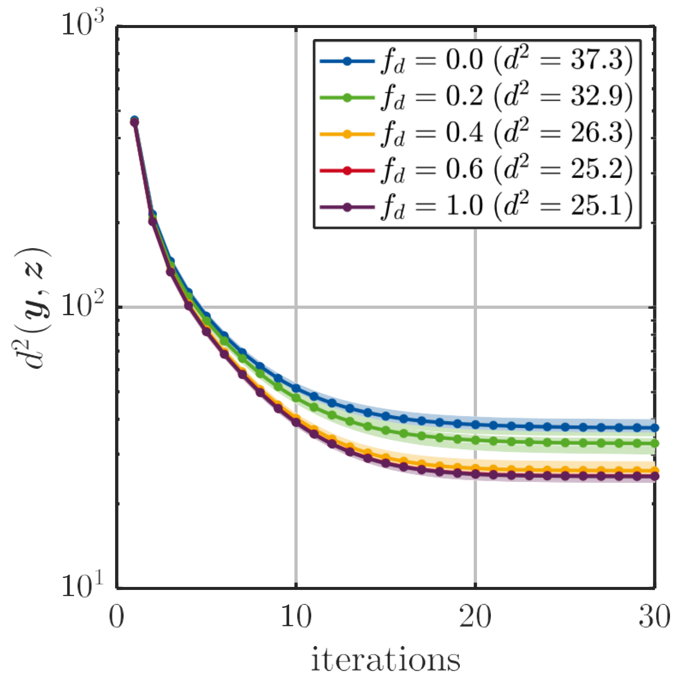
- *Accuracy–speed tradeoff*
- Initial iterations: Low query accuracy, minimize backtracking
- Final iterations: High query accuracy, maximize backtracking

Benchmark problem: Torsion of cube



- DD nonlinear-elastic cube
- 20x20x20= 8000 elements
- 64,000 material (Gauss) points
- *1 million material data points*

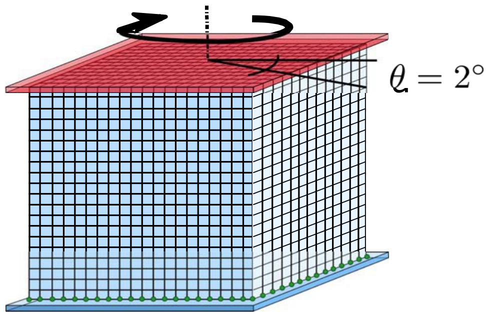




Effect of backtracking on *k-means search* speed and accuracy
(f_d = 0: no backtracking; f_d = 1: full backtracking)

R. Eggersmann, L. Stainier, M. Ortiz & S. Reese, *CMAME*, **382** (2021) 113855.

Benchmark problem: Torsion of cube



- DD nonlinear-elastic cube
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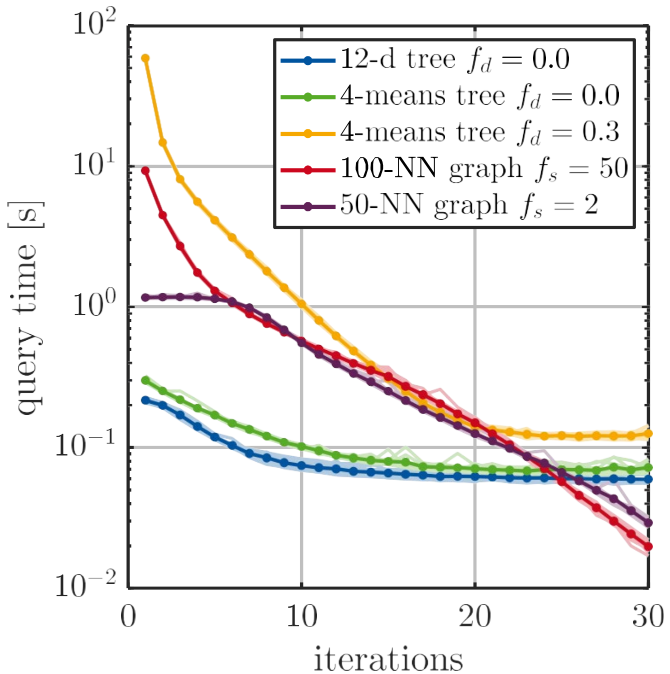
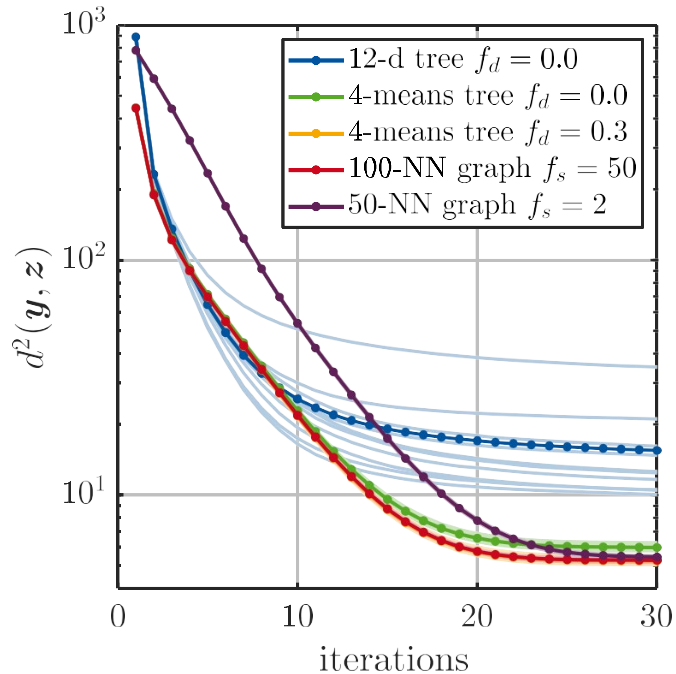
- *100 million material data points*

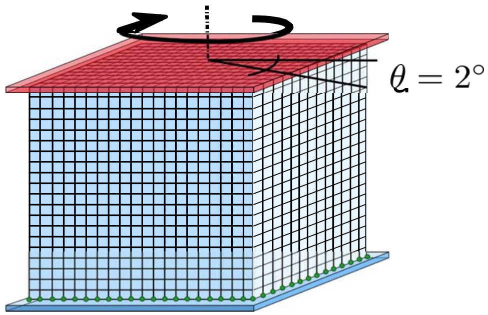




Comparison of search algorithms:
kd-tree, k-means and kNN graph algorithms

R. Eggersmann, L. Stainier, M. Ortiz & S. Reese, *CMAME*, **382** (2021) 113855.

Benchmark problem: Torsion of cube



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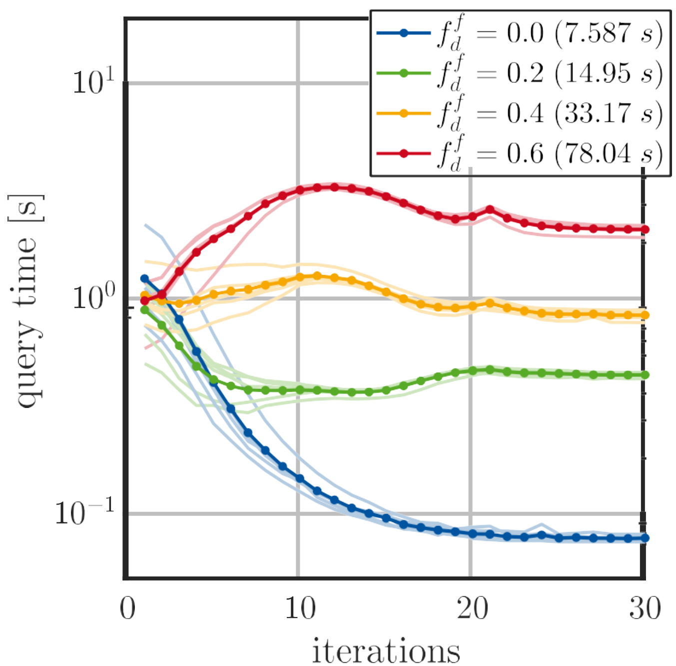
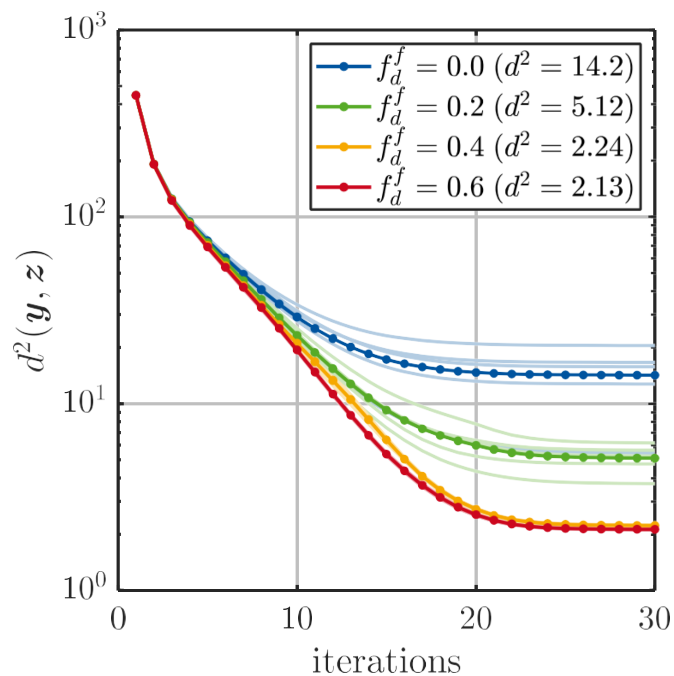
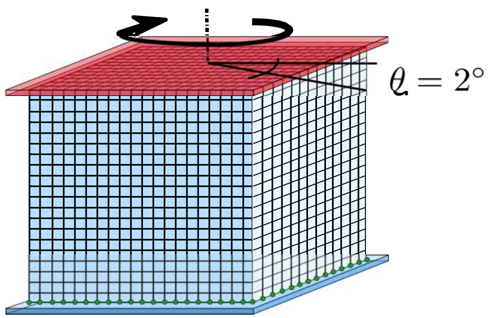




k-means with different degrees of backtracking.
search accuracy set to increase with DD iteration

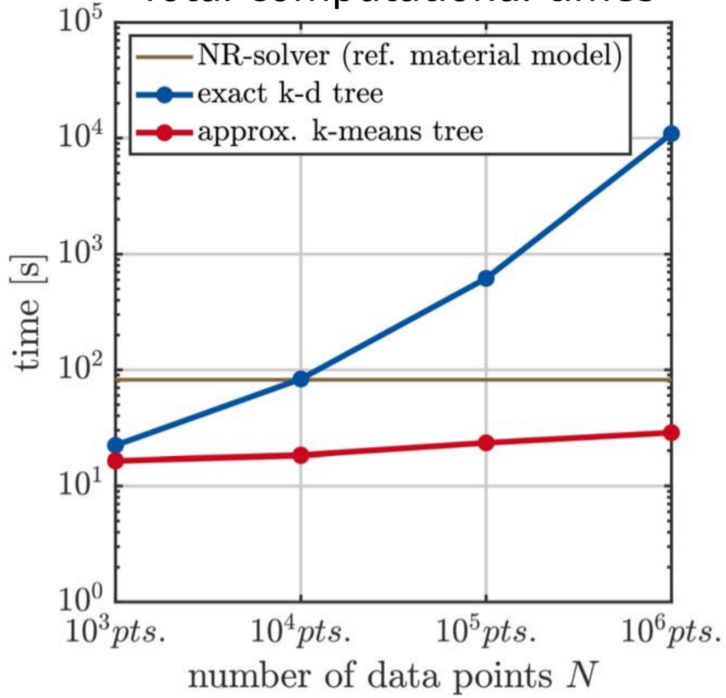
R. Eggersmann, L. Stainier, M. Ortiz & S. Reese, *CMAME*, **382** (2021) 113855.

Benchmark problem: Torsion of cube



- DD nonlinear-elastic cube
- 20x20x20= 8000 elements
- 64,000 material (Gauss) points
- 10^3-10^6 material data points

Total computational times

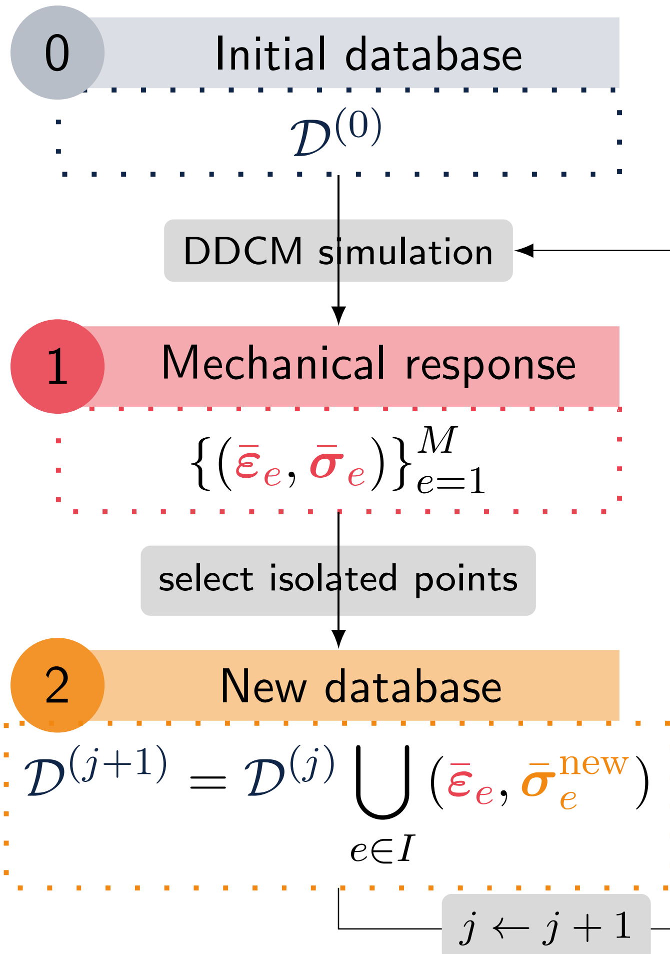


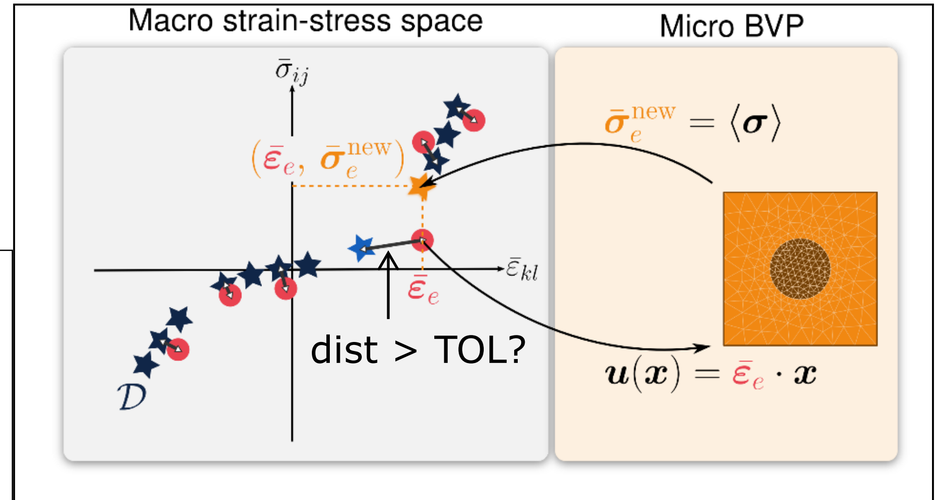
- *DD solver beats Newton-Raphson!*
- DD requires the solution of linear FE problems only (projection P_E to the admissible set E)
- System matrix can be factorized once and for all at the outset
- Load increments require back-substitution operations only
- Newton-Raphson requires repeated assembly and factorization of stiffness matrix
- *DD factorization advantage offsets data-searching overhead!*

R. Eggersmann, L. Stainier, M. Ortiz & S. Reese, *CMAME*, **382** (2021) 113855.

Need data!
(adaptive learning)

Data-Driven computing with Adaptive Learning



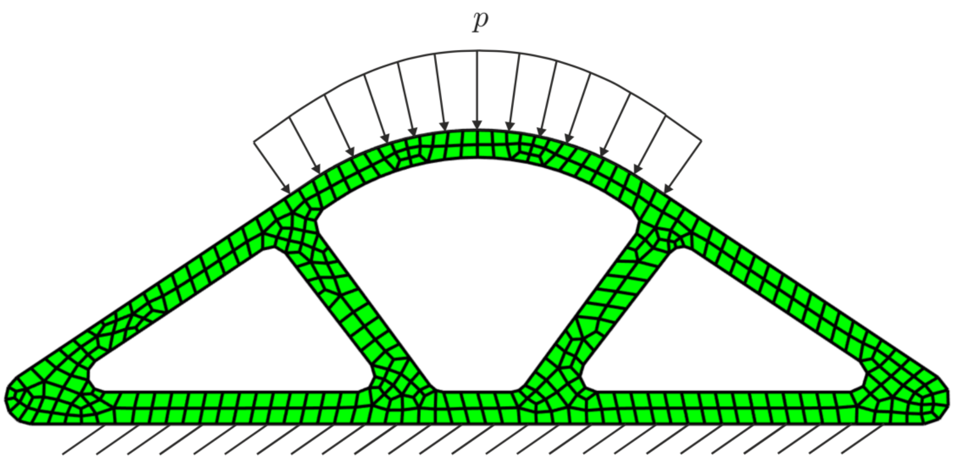


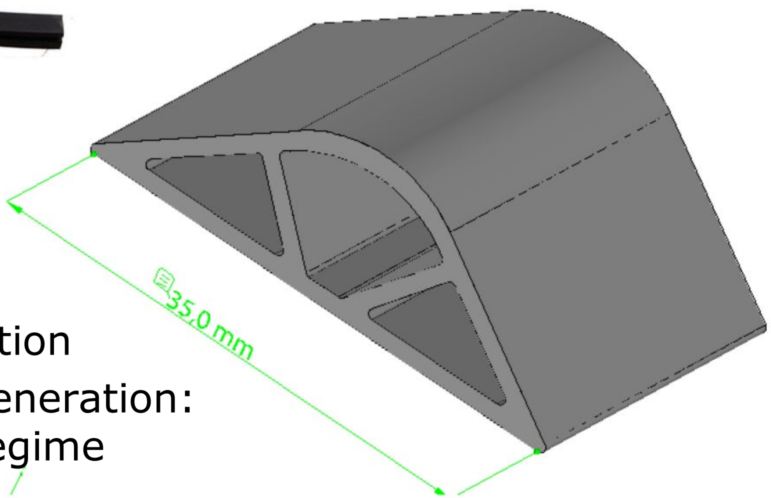
- *Generate new data from RVE when local state dist(z_e ,D) > TOL!*
- Areas of *poor coverage* in the material data set are detected by DD solver, filled in *on-the-fly* as needed
- Material data set adapted to solution (*goal-oriented adaptive learning*)
- *Concurrent multiscale analysis* with *material data re-use* (unlike FE^2)

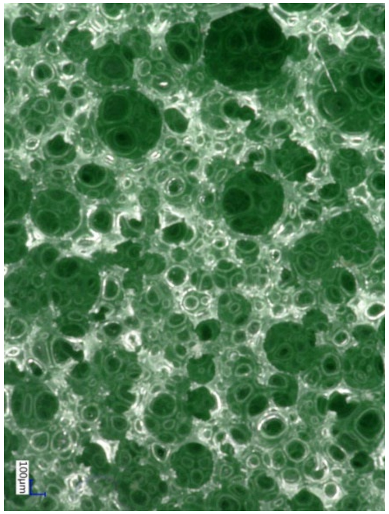
A. Platzer, Ph.D. Dissertation, l'Ecole Centrale de Nantes, 2020.

Adaptive learning: Open-cell rubber foams

- Application: *Rubber sealing*
- Sealing for doors, windows
- Self adhesive, loaded by pressure







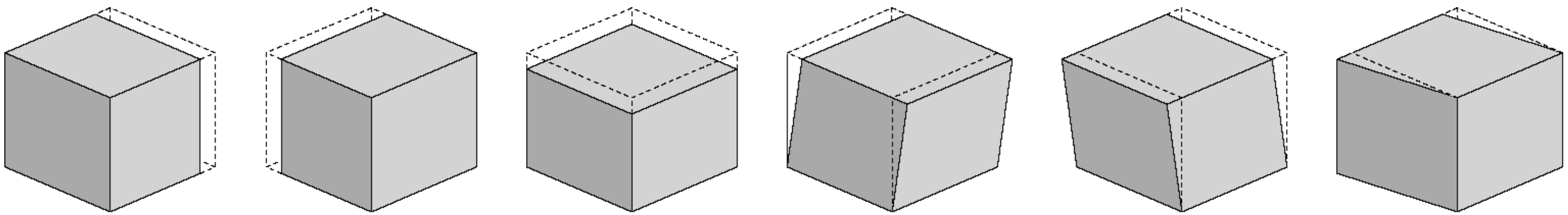
microstructure

- Open-cell foam
- Isotropic material
- 3D analysis/simulation
- Microscopic data generation: linear/non-linear regime

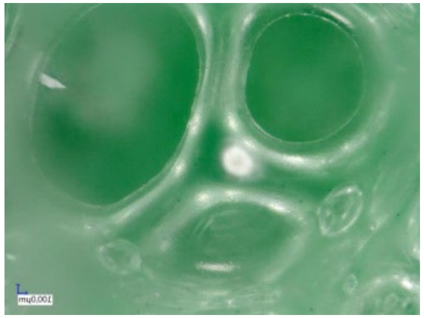
T.F. Korzeniowski and K. Weinberg, *CMAME*, **400** (2022) 115487.

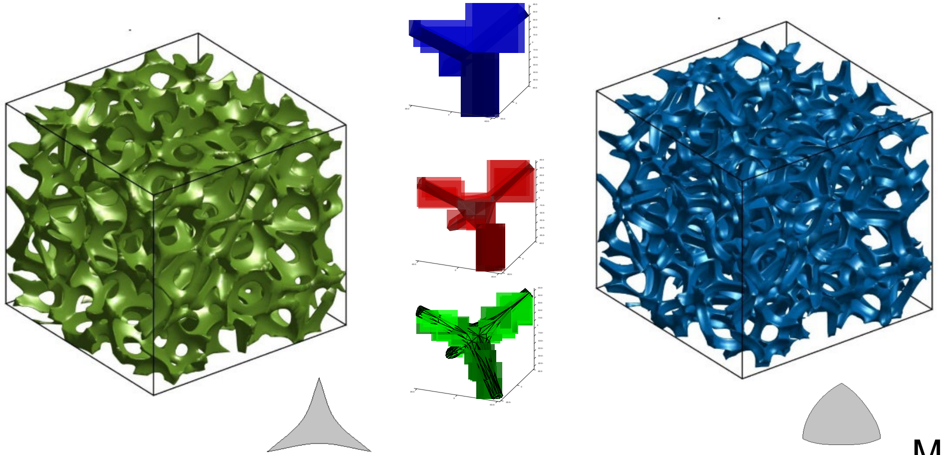
Adaptive learning: Open-cell rubber foams

- Data sets: $\mathcal{D} = \{(\epsilon, \sigma)\}$, or $\mathcal{D} = \{(F, P)\}$, or $\mathcal{D} = \{(C, S)\}$.
  1. apply deformation
  2. compute RVE and determine average stress
  3. collect data pairs
- *Can generate macroscopic data on demand as required!*
- Six different loading cases (unit loads)



- 3D open-cell foam RVEs:

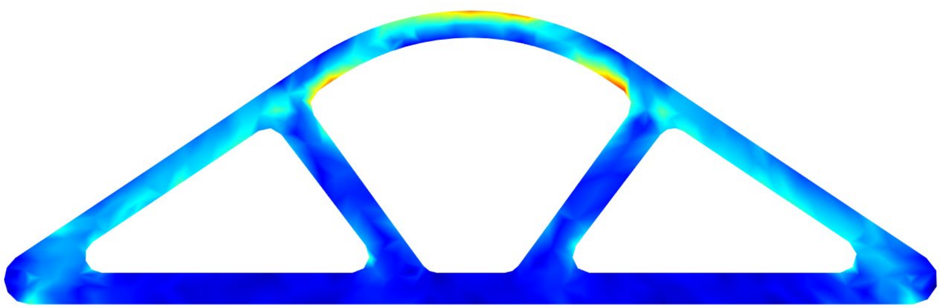


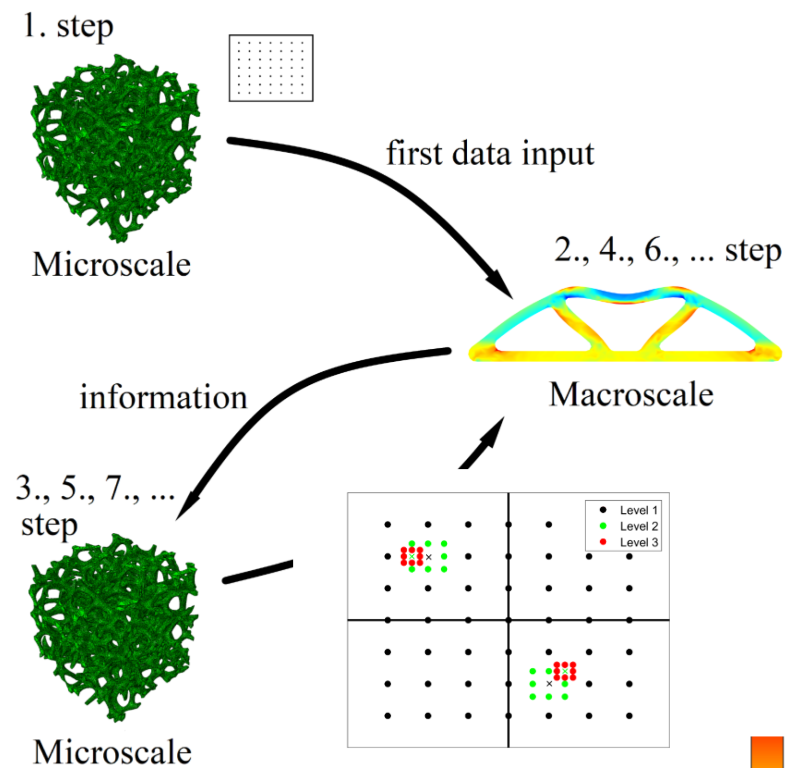


T.F. Korzeniowski and K. Weinberg, *CMAME*, **400** (2022) 115487.

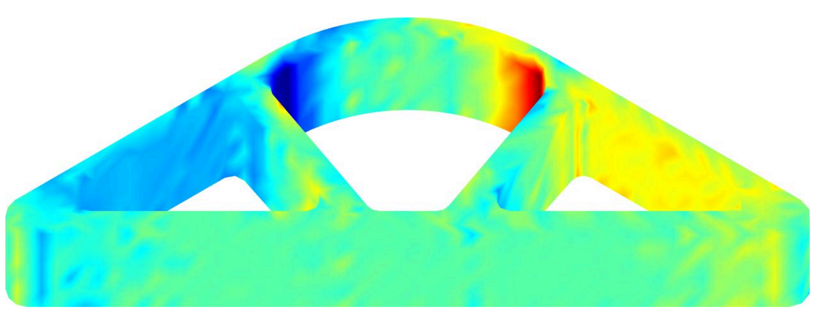
Adaptive learning: Open-cell rubber foams

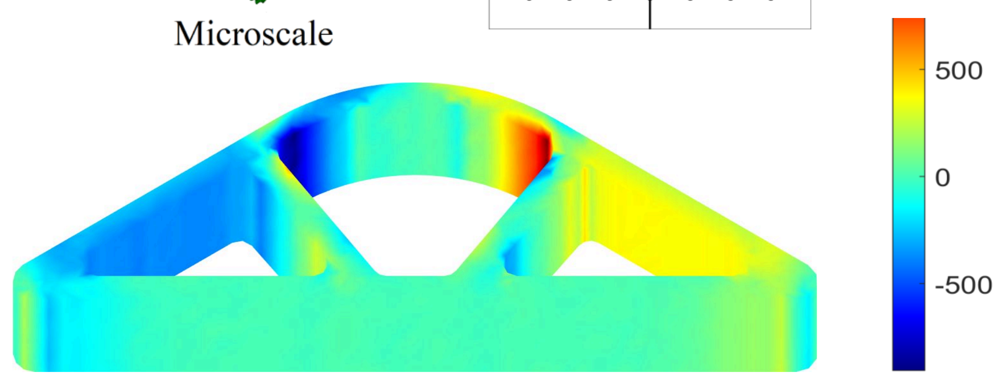
- Cauchy stress distribution (vertical)
- Start with a coarse level of data
- Identify data points of interest
- Do additional RVE calculations
- Redo computation at finer level till TOL





- Shear stress at level 1 and level 4



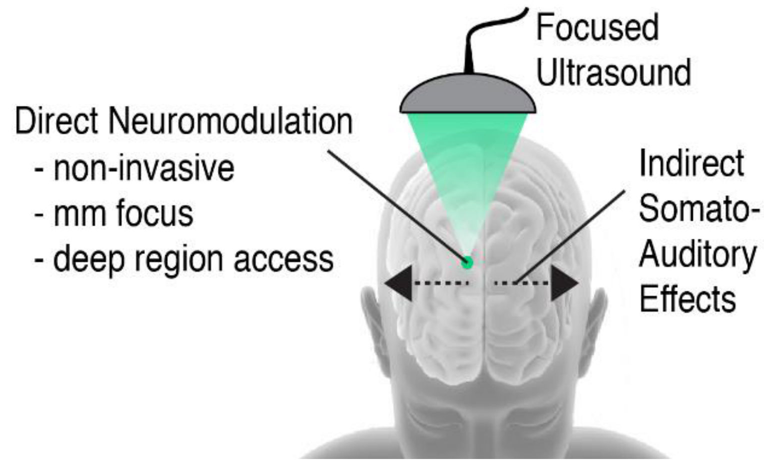


T.F. Korzeniowski and K. Weinberg, *CMAME*, **400** (2022) 115487.

Application: Ultrasound neuromodulation

Towards DD patient-specific UNM

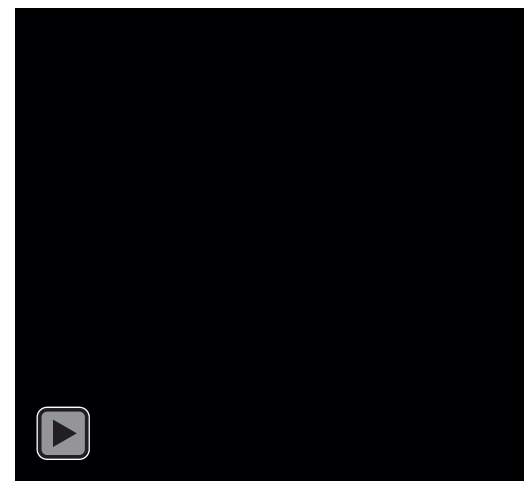
- *Ultrasonic neuromodulation* (UNM) is a novel non-invasive technique that uses *low intensity focused ultrasound* (LIFU) to stimulate the brain.
- First proposed in 2002 by A. Bystritsky as possibly having therapeutic benefits.
- W. Tyler and team discovered UNM is able to stimulate high neuron activity.
- UNM is currently used clinically to treat neurological disorders and improving cognitive function.
- Optimizing UNM therapies in a clinical setting requires *advanced patient-specific data-acquisition and simulation capability*.



Bystritsky A., USPTO patent 7,283,861, 2002.

Tyler, W.J., Tufail, Y., Finsterwald, M., Tauchmann, M.L., Olson, E.J., Majestic, C., PLoS One. 2008;3(10):e3511.

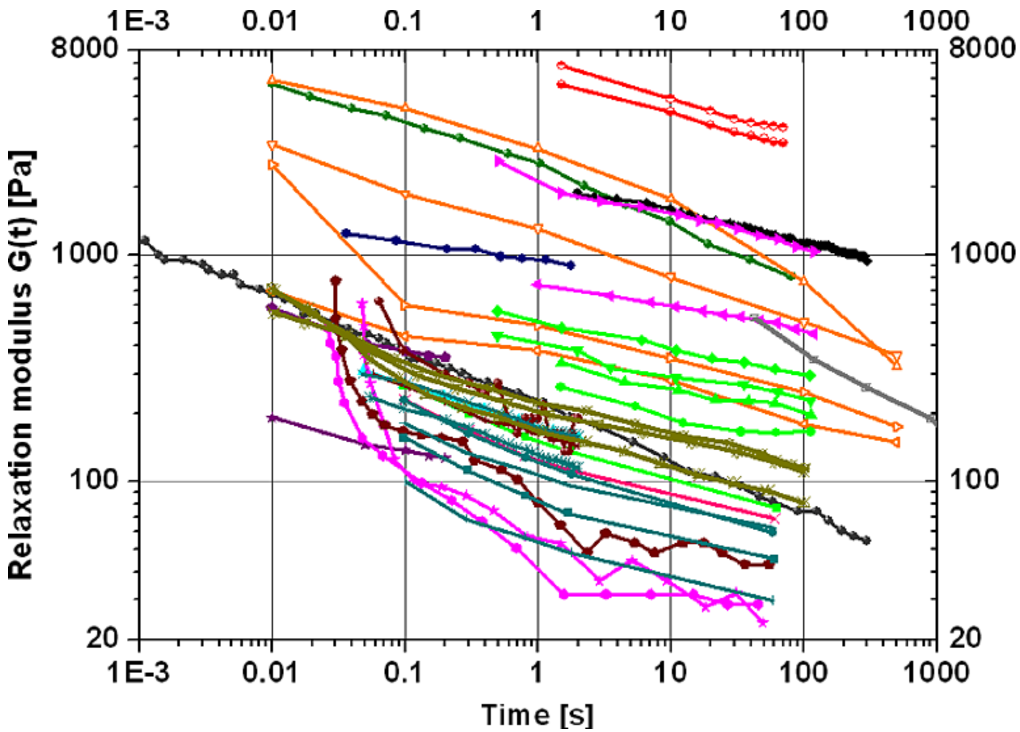
Salahshoor, S., Shapiro, M. and Ortiz, M. Appl. Phys. Lett. **117**, 033702 (2020)



3D FE simulation of Pressure waves under Transcranial LIFUS (100 kHz).

Towards DD patient-specific UNM

- *Complexity* and *variability* of brain viscoelasticity defy effective *ad hoc* modeling!



In vitro relaxation modulus versus time from literature survey. Curves were obtained from either compression or shear quasi-static experiments.

Chatelin, S., Constantinesco, A., Willinger, R., "Fifty years of brain tissue mechanical testing: from in vitro to in vivo investigations." *Biorheology*. 2010;**47**(5-6):255-76.

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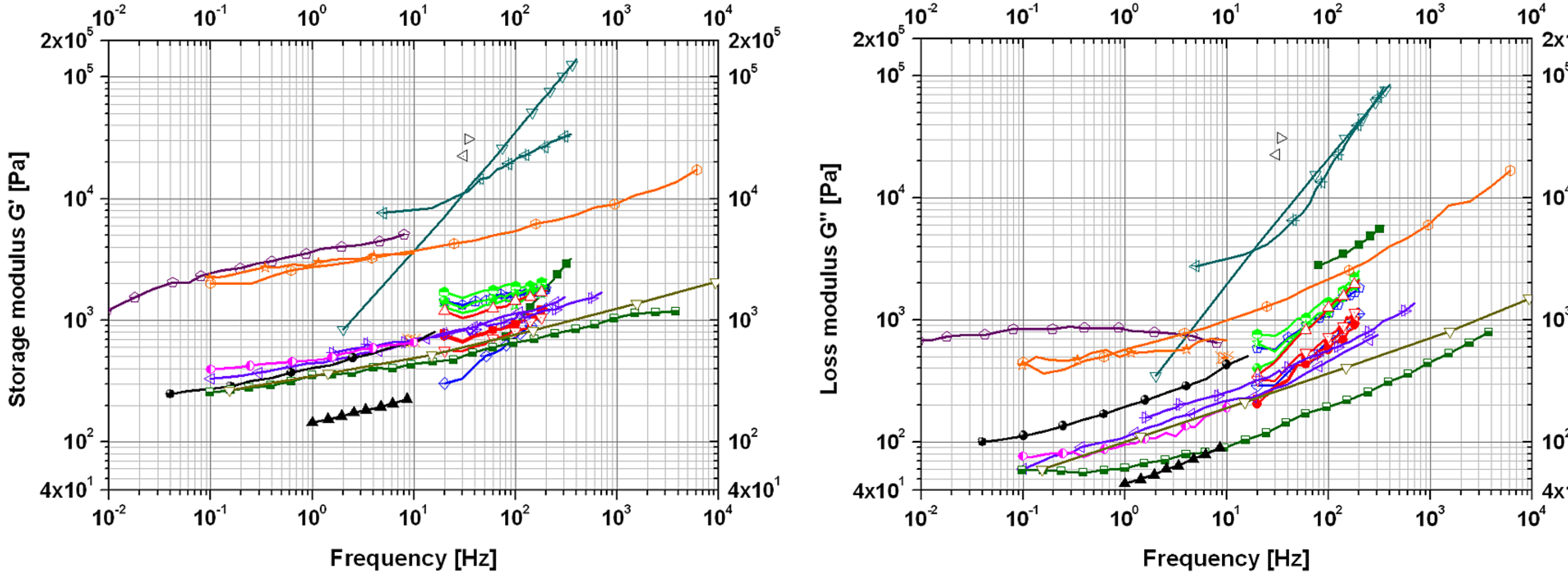


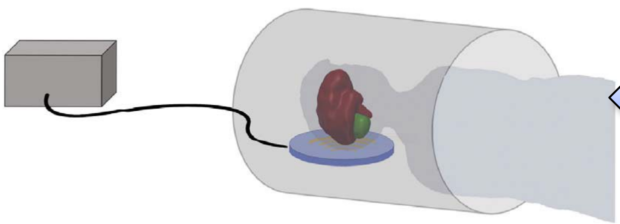
Storage and loss moduli of brain tissue compiled from literature survey of *in vitro* dynamic frequency sweep tests in shear.

Chatelin, S., Constantinesco, A., Willinger, R., "Fifty years of brain tissue mechanical testing: from in vitro to in vivo investigations." *Biorheology*. 2010;**47**(5-6):255-76.

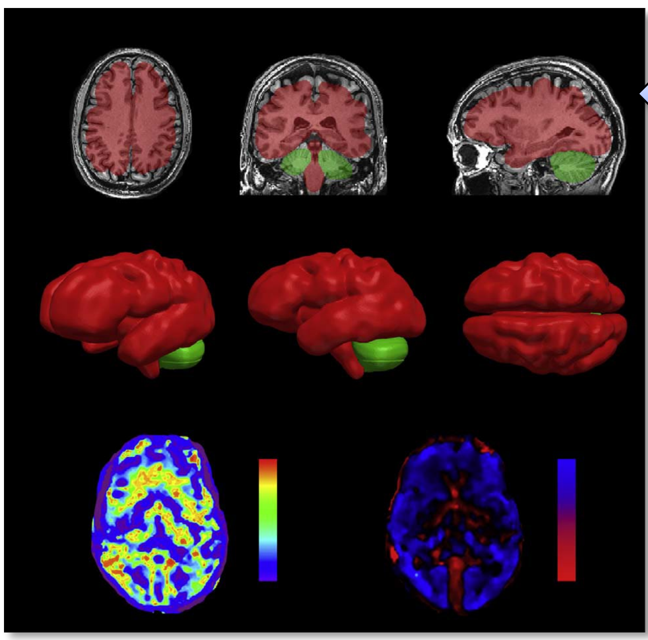
Michael Ortiz
EC25 10/25/22

Towards DD patient-specific UNM

- Data can be acquired *in vivo* through Magnetic Resonance Elastography (EMR).
- MRE is based on the magnetic resonance imaging of shear wave propagation.

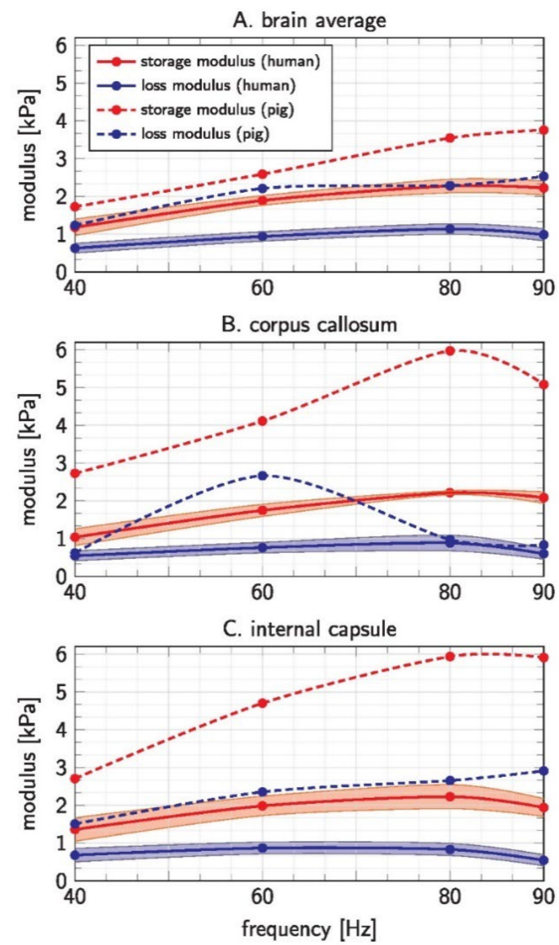


Human subjects scanned in the supine position



Structural scan, reconstruction map of storage and loss moduli

Region-specific storage and loss moduli for human and porcine brains as functions of driving frequency



J. Weickenmeier, M. Kurt, E. Ozkaya, M. Wintermark, K.B. Pauly, E. Kuhl, *J. Mech. Behav. Biomed. Mater.*, **77** (2018) 702-710.

Model-free Data-Driven viscoelasticity

- Field equations,

$$\epsilon_e(t) = B_e u(t) + g_e(t), \quad e = 1, \dots m,$$

$$M\ddot{u}(t) + \sum_{e=1}^{m} w_e B_e^T \sigma_e(t) = f(t).$$

- Fourier-transform representation,

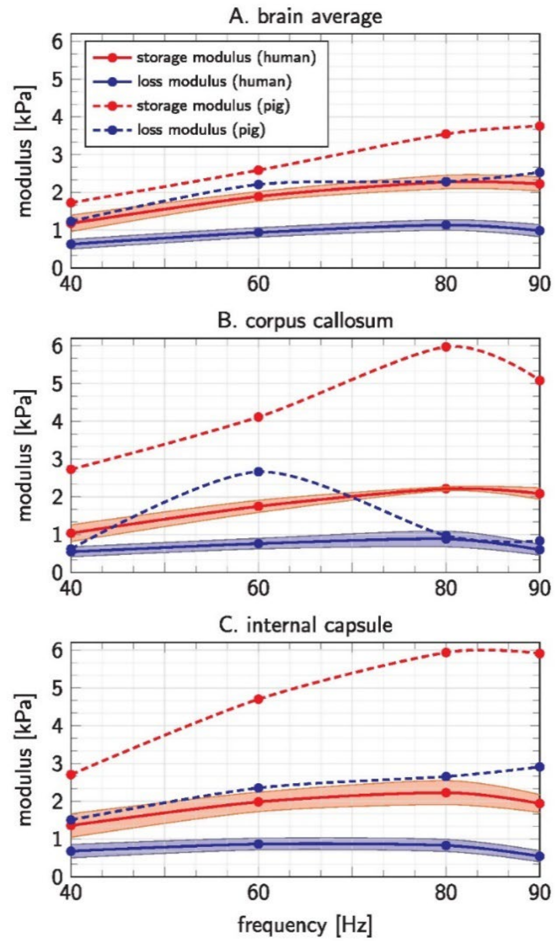
$$\hat{\epsilon}_e(\omega) = B_e \hat{u}(\omega) + \hat{g}_e(\omega), \quad e = 1, \dots m,$$

$$\sum_{e=1}^{m} w_e B_e^T \hat{\sigma}_e(\omega) - M\omega^2 \hat{u}(\omega) = \hat{f}(\omega).$$

- Complex-modulus: $\hat{\sigma}_e(\omega) = \mathbb{E}(\omega)\hat{\epsilon}_e(\omega)$.

- Displacement problem: For every $\omega \in \mathbb{R}$,

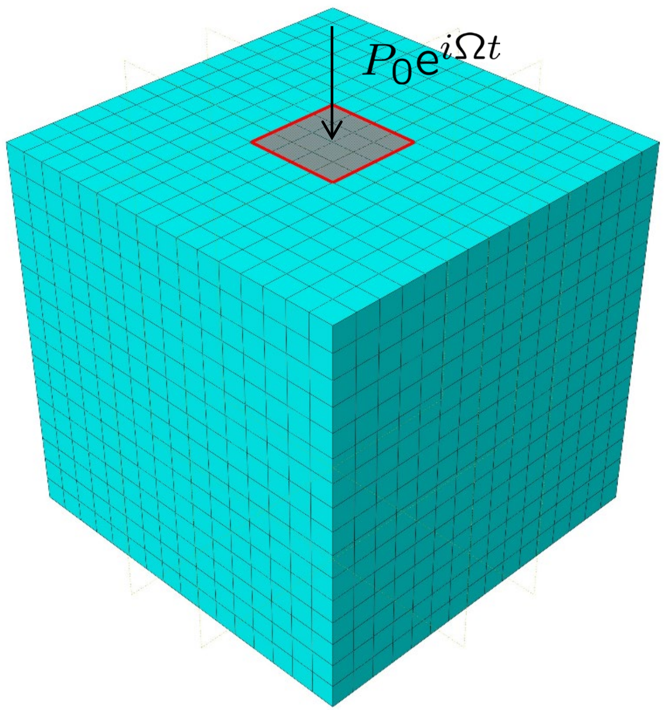
$$\sum_{e=1}^{m} w_e B_e^T \mathbb{E}(\omega)(B_e \hat{u}(\omega) + \hat{g}_e(\omega)) - M\omega^2 \hat{u}(\omega) = \hat{f}$$



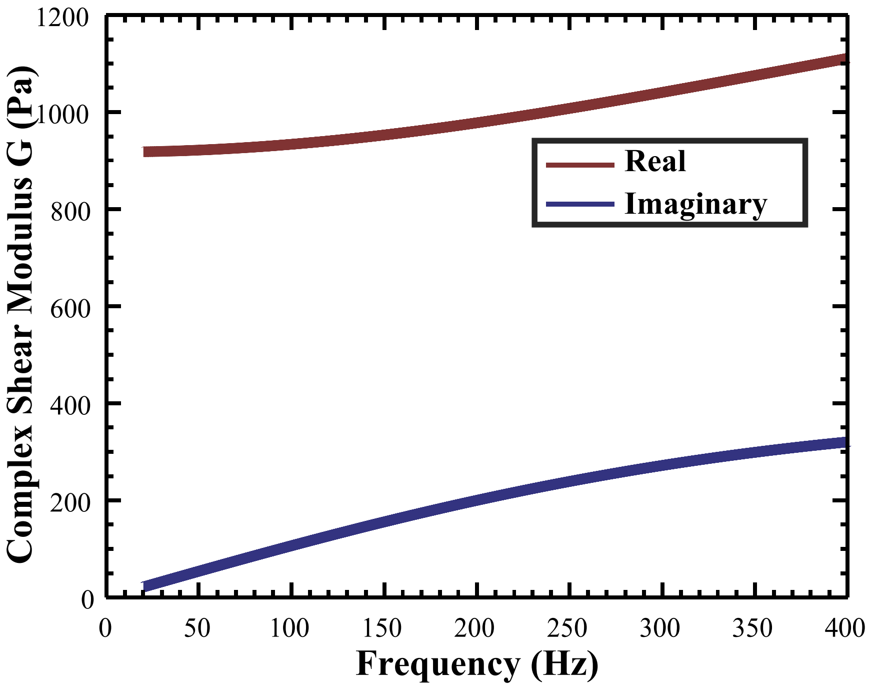
J. Weickenmeier, M. Kurt, E. Ozkaya, M. Wintermark, K.B. Pauly, E. Kuhl, *J. Mech. Behav. Biomed. Mater.*, **77** (2018) 702-710.

Model-free Data-Driven viscoelasticity

- Test of convergence: *Insonated agarose gel block*.



Finite-element discretization of agarose gel block into hex-elements. The square on the top surface marks area of application of harmonic pressure



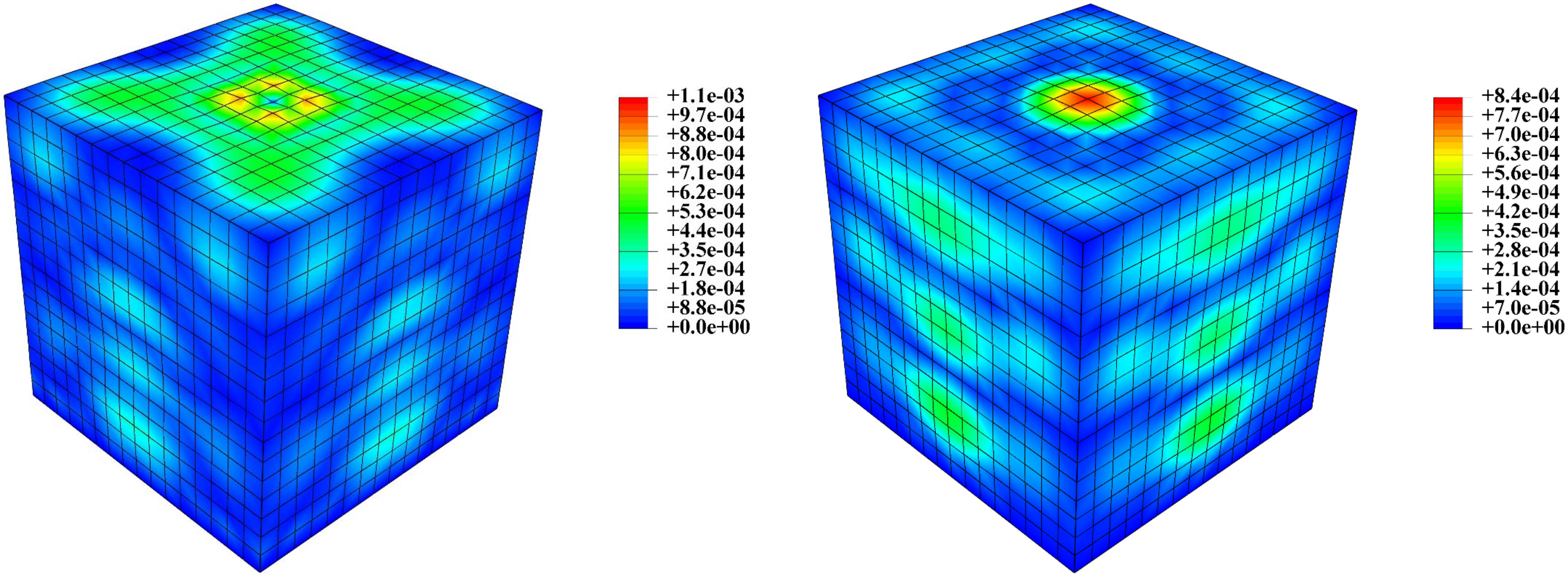
Complex moduli of agarose gel measured using dynamic shear testing (DST) and magnetic resonance elastography (MRE).

R. J. Okamoto, E. H. Clayton and P. V. Bayly, Physics in Medicine & Biology 56 (19) (2011) 6379.

H. Salahshoor and M. Ortiz, arXiv 2205.06674v1, May 13, 2022.

Model-free Data-Driven viscoelasticity

- Test of convergence: *Insonated agarose gel block*.



Insonated agarose gel block.
Displacements for applied frequency Ω = 1000 Hz.
a) Real component. b) Imaginary component.

H. Salahshoor and M. Ortiz, arXiv 2205.06674v1, May 13, 2022.

Michael Ortiz
EC25 10/25/22

Model-free Data-Driven viscoelasticity

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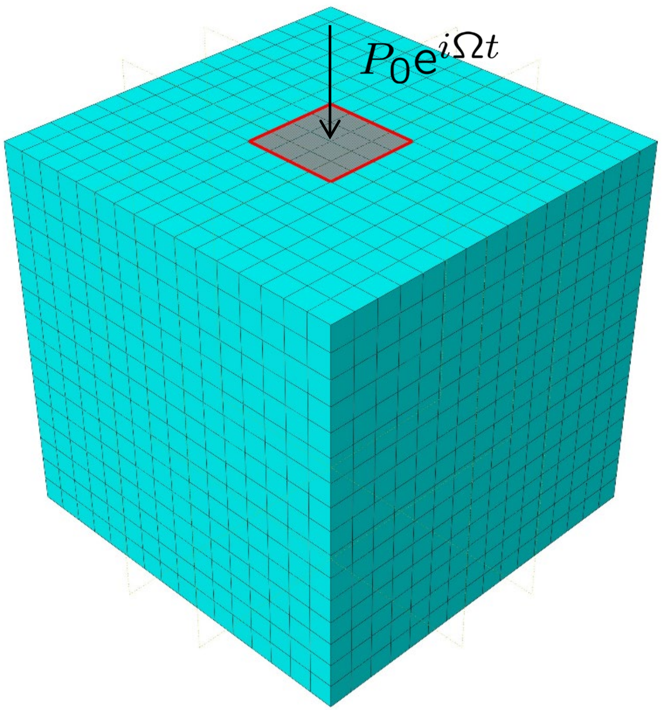
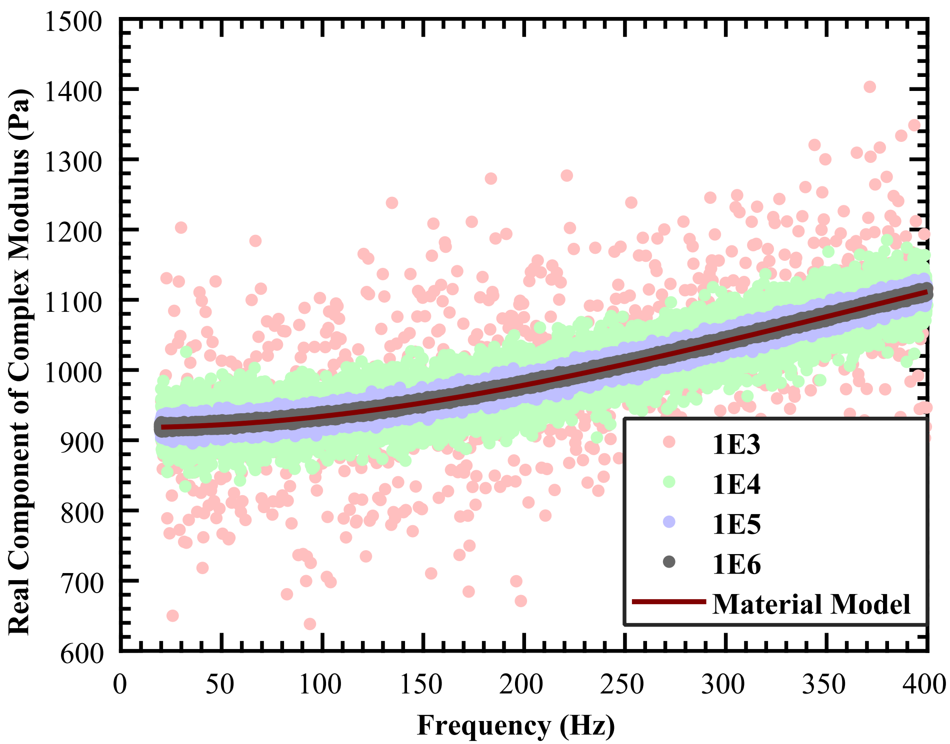


Finite-element discretization of agarose gel block into hex-elements. The square on the top surface marks area of application of harmonic pressure



Data of sizes 10^3, 10^4, 10^5 and 10^6 used in the DD calculations. Real component of complex modulus.

H. Salahshoor and M. Ortiz, arXiv 2205.06674v1, May 13, 2022.

Model-free Data-Driven viscoelasticity

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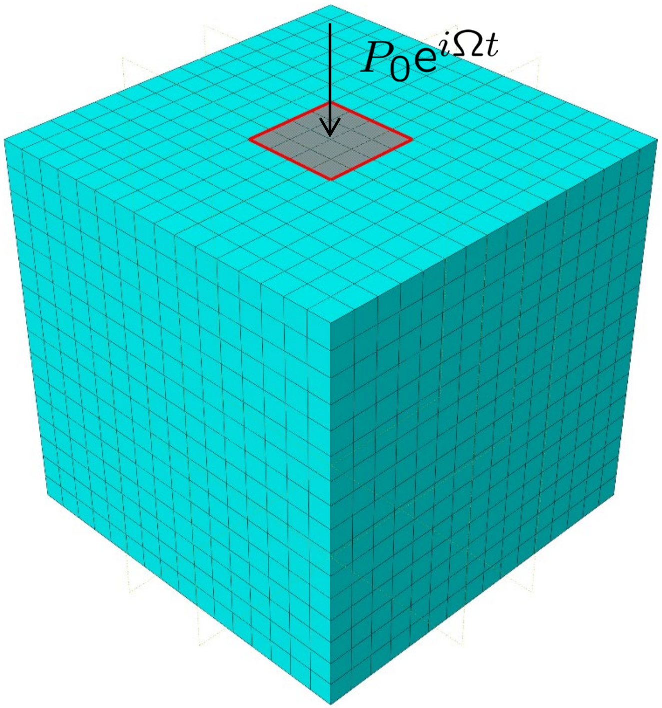

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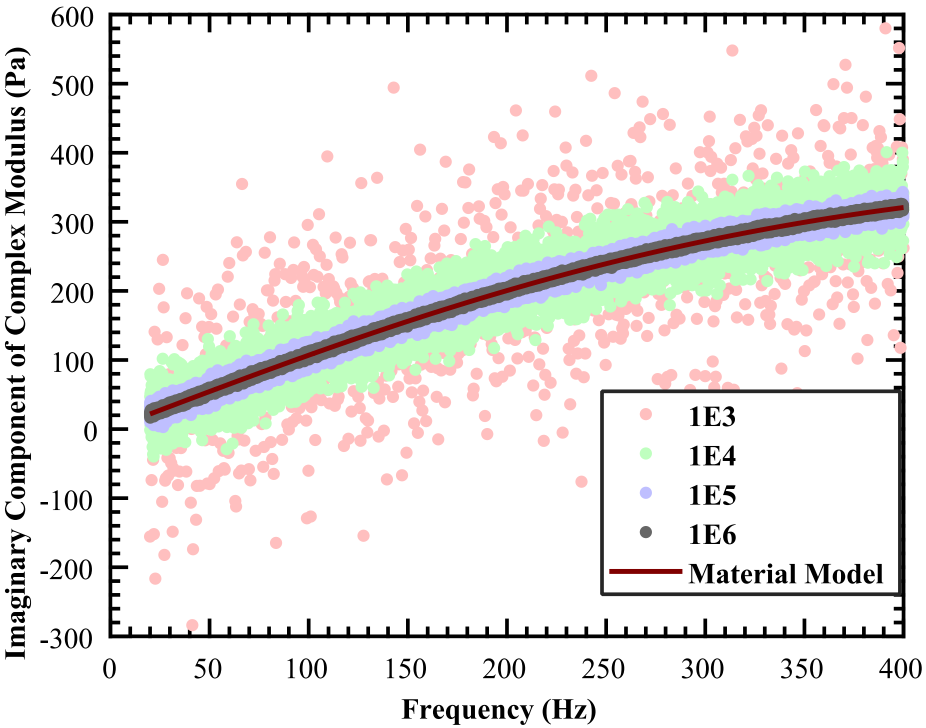

Data of sizes 10^3, 10^4, 10^5 and 10^6 used in the DD calculations. Imaginary component of complex modulus.

H. Salahshoor and M. Ortiz, arXiv 2205.06674v1, May 13, 2022.

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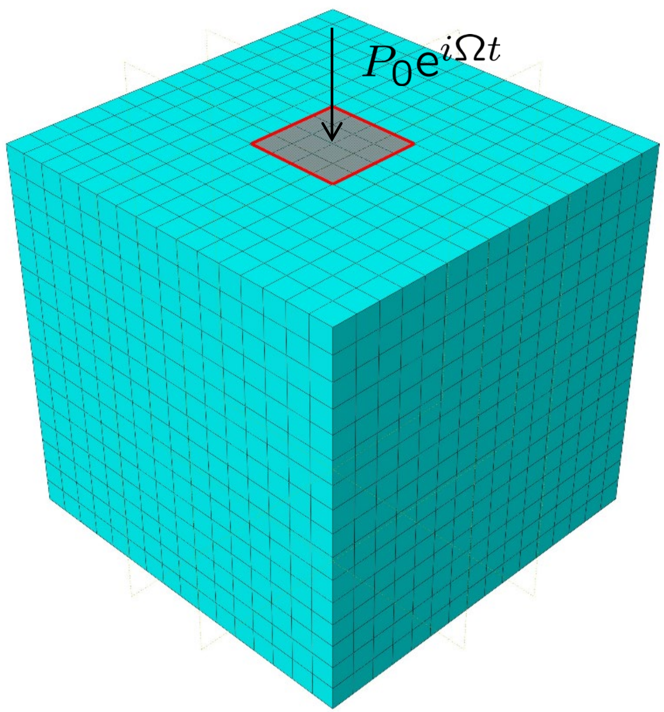

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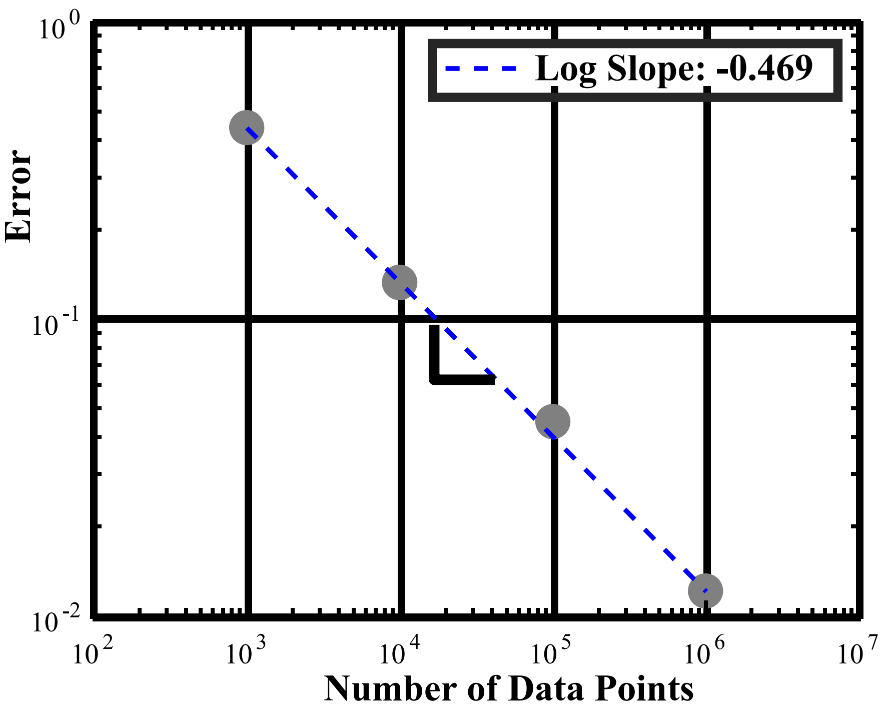
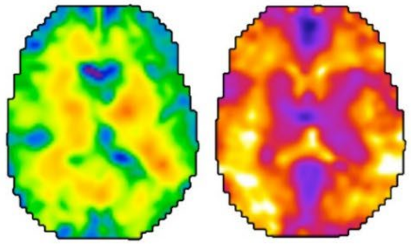


Normalized flat-norm convergence error as a function of the number of data points, showing a clear trend towards convergence.

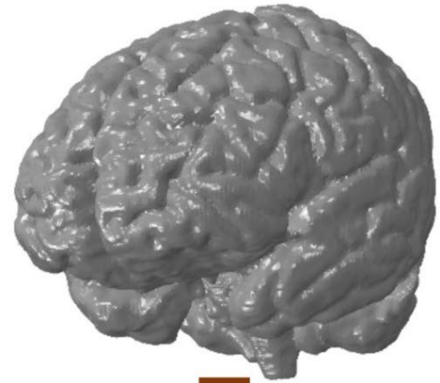
H. Salahshoor and M. Ortiz, arXiv 2205.06674v1, May 13, 2022.

Towards DD patient-specific UNM

Experiment:
In-situ data acquisition (MRI & MRE)

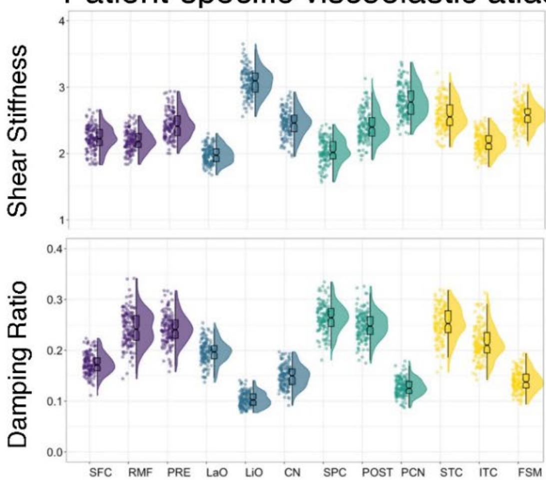


Anatomical Data

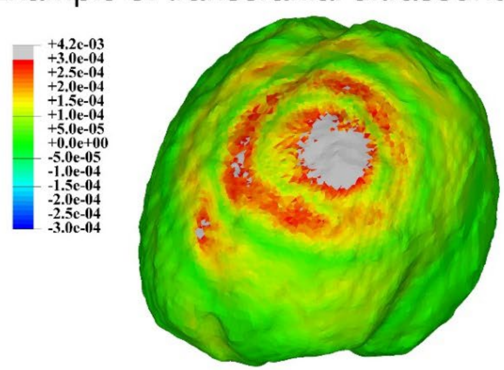


MFDD
UNM

Brain Matter Data Repository:
Patient-specific viscoelastic atlas



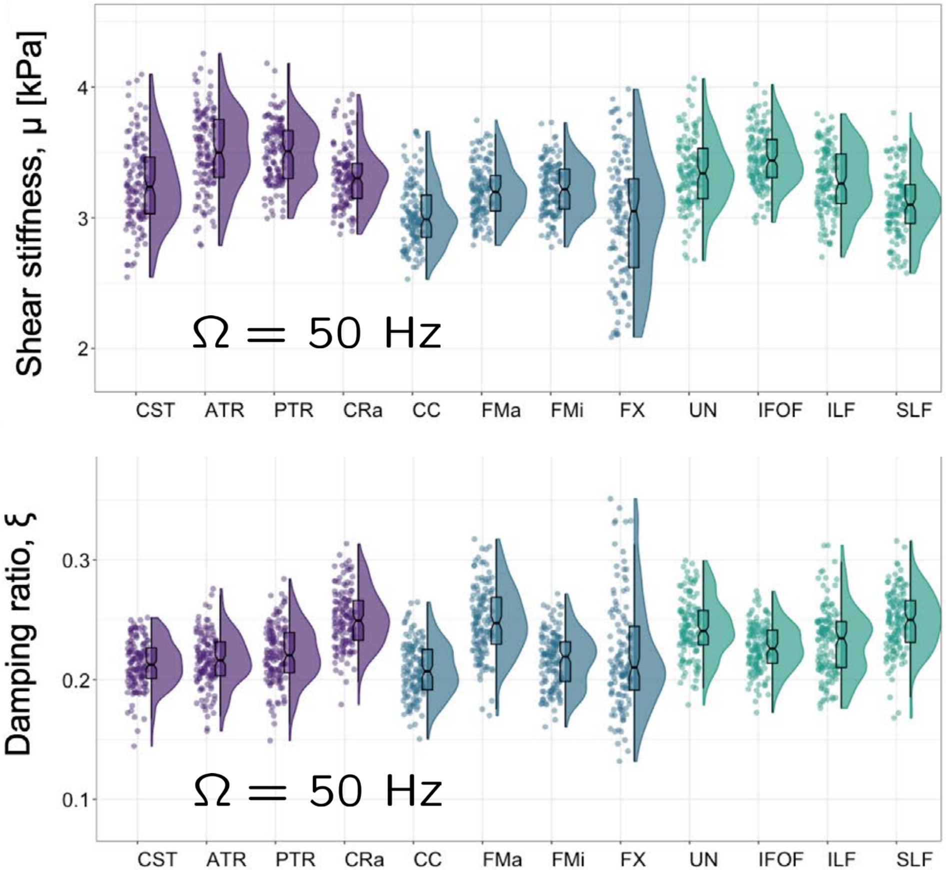
Direct Data-Driven patient-specific prediction of wave patterns in brain:
Example of transcranial ultrasound



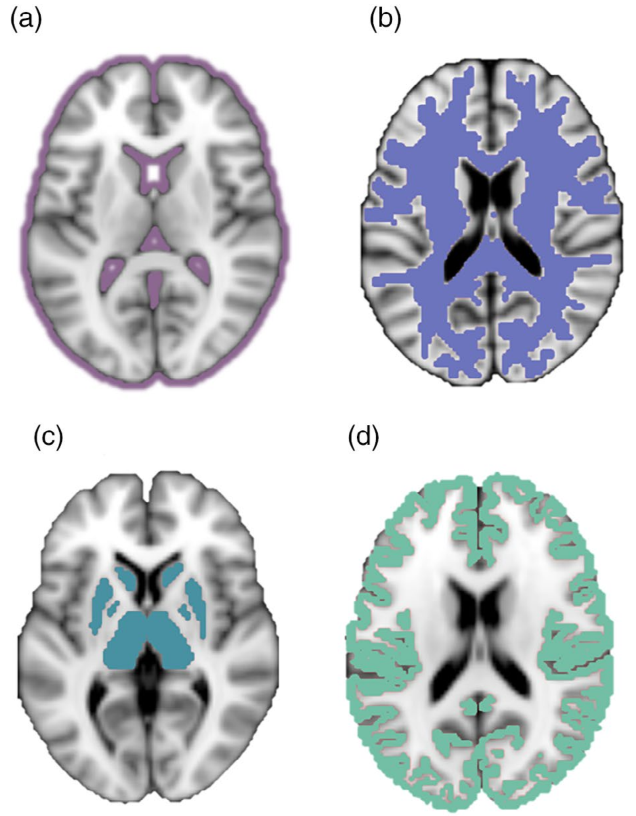
L.V. Hiscox *et al.*, *Hum Brain Mapp.*, 2020;**41**:5282–5300
H. Salahshoor and M. Ortiz, bioRxiv 2022.09.01.506248, Sept 1, 2022.

Towards DD patient-specific UNM

- Data can be acquired *in vivo* through Magnetic Resonance Elastography (MRE) .
- MRE is based on the magnetic resonance imaging of shear wave propagation.



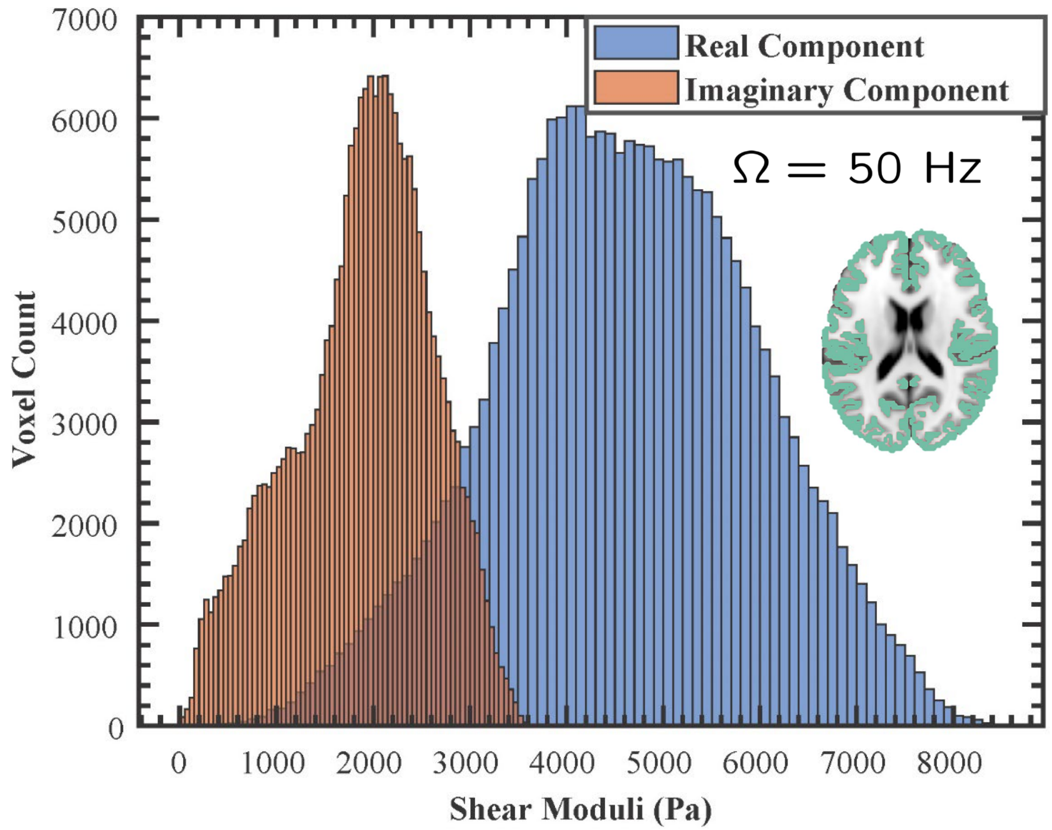
MRE viscoelastic data atlas at 12 regions of interest (Desikan–Killiany–Tourville cortical labelling protocol).



MRE data for (a) the entire brain (b) white matter (c) subcortical gray matter (d) cerebral cortex.

H. Salahshoor and M. Ortiz, bioRxiv 2022.09.01.506248, Sept 1, 2022.

Towards DD patient-specific UNM



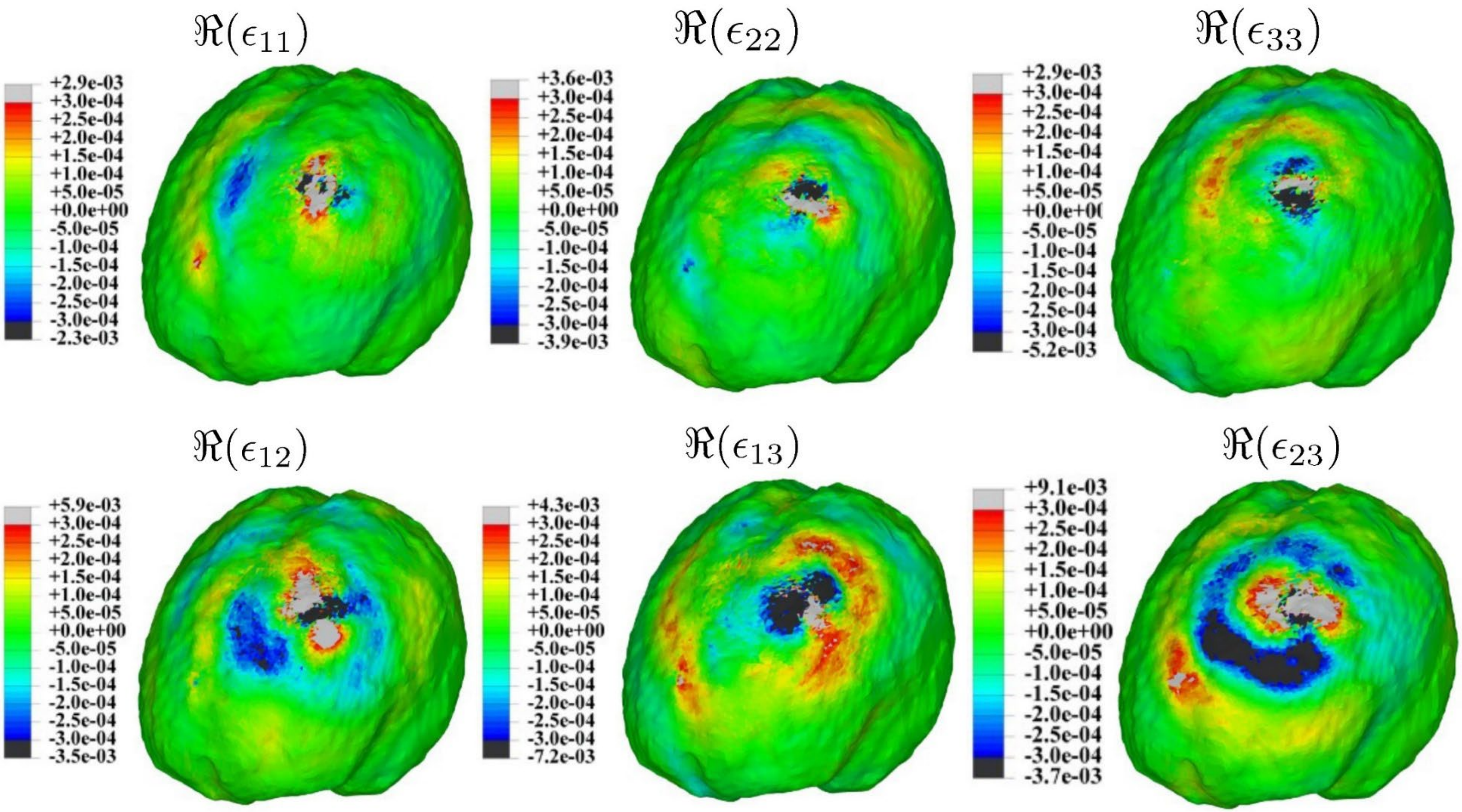
Finite element model of human brain reconstructed from MRI data, 0.2 million tetrahedral elements. Transcranial stimulation is modeled by subjecting the highlighted region to harmonic pressure as a traction boundary condition.

Histogram of complex moduli from in vivo MRE data. The finite-element model is co-registered to the MRE data.

L.V. Hiscox *et al*., *Hum Brain Mapp*., 2020;**41**:5282–5300

H. Salahshoor and M. Ortiz, bioRxiv 2022.09.01.506248, Sept 1, 2022.

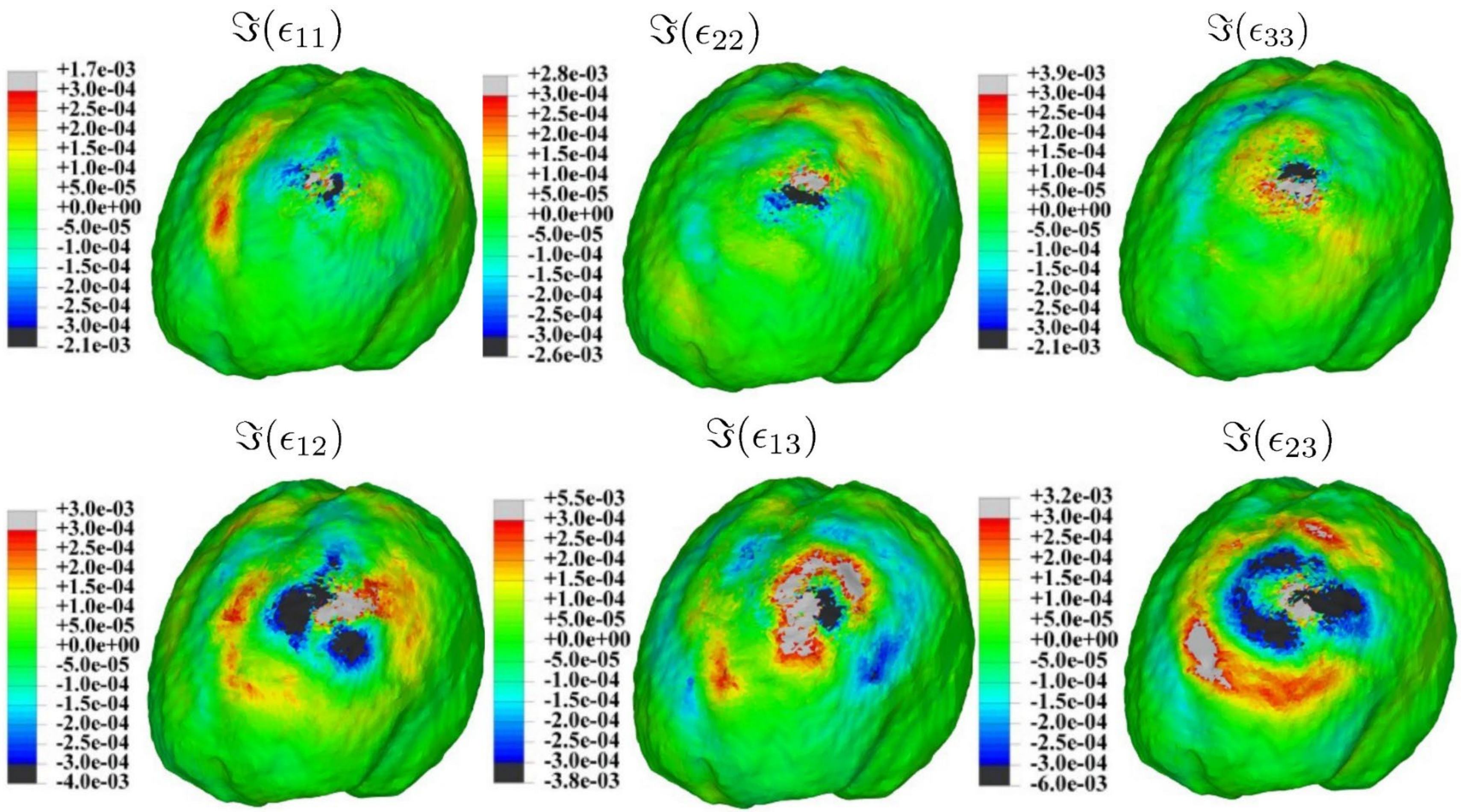
Towards DD patient-specific UNM



Model-free Data-Driven simulation of transcranial harmonic stimulation on a region on the frontal lobe of the brain at Ω = 50 Hz.
Real part of the strain field components at steady state.

H. Salahshoor and M. Ortiz, bioRxiv 2022.09.01.506248, Sept 1, 2022.

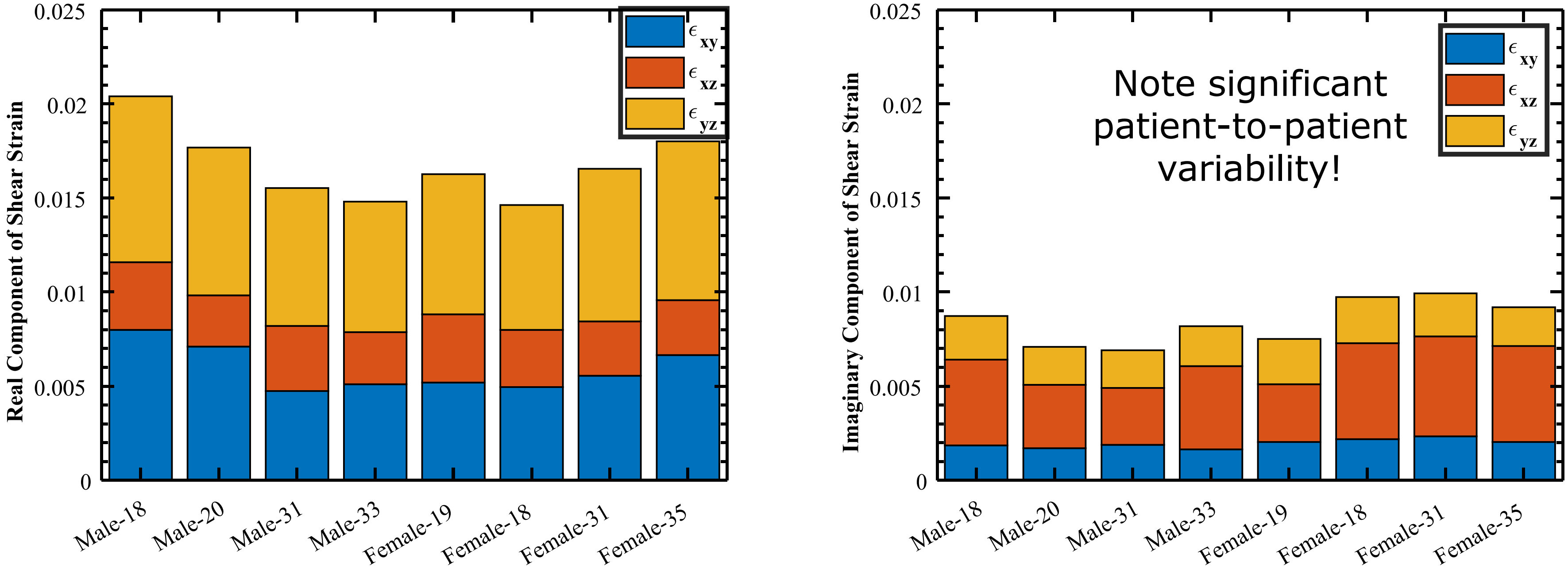
Towards DD patient-specific UNM



Model-free Data-Driven simulation of transcranial harmonic stimulation on a region on the frontal lobe of the brain at Ω = 50 Hz. Imaginary part of the strain field components at steady state.

H. Salahshoor and M. Ortiz, bioRxiv 2022.09.01.506248, Sept 1, 2022.

Towards DD patient-specific UNM



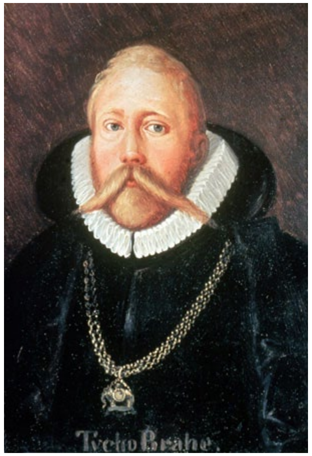
Model-free Data-Driven simulation of transcranial harmonic stimulation on a region on the frontal lobe of brain at Ω = 50 Hz, for eight patient-specific MRE data sets. Real and imaginary maximum strain amplitudes at steady state.

- NB: Improvements to MRE required to extend the technology to the *ultrasound range*, currently under development.
- *Model-Free Data-Driven viscoelasticity* provides a path for the direct on-the-fly integration of *in vivo* patient-specific data into calculations supporting future UNM clinical applications!

H. Salahshoor and M. Ortiz, bioRxiv 2022.09.01.506248, Sept 1, 2022.

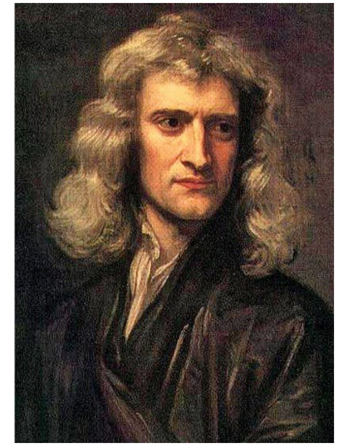
What have we learned?
(not much…)

Empirical vs. epistemic knowledge

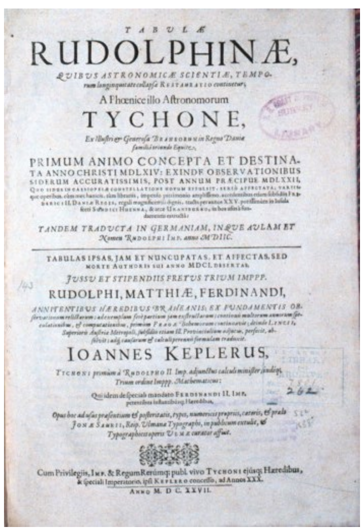


Tycho Brahe
(1546-1601)

Johannes Kepler
(1571-1630)



Sir Issac Newton
(1643-1727)



TABULÆ
RUDOLPHINÆ,
QUIBUS ASTRONOMICÆ SCIENTIÆ, TEMPO-
A Phœnice illo Astronomorum
TYCHONE,
PRIMUM ANIMO CONCEPTA ET DESTINA-
TA ANNO CHRISTI MDLXIV: EXINDE OBSERVATIONIBUS
SIDERUM ACCURATISSIMIS, POST ANNUM PRÆCIPUE MDLXXII,
TANDEM TRADUCTA IN GERMANIAM, INQUE AULAM ET
Nomen RUDOLPHI IMP. anno MDIIC.
TABULAS IPSAS, JAM ET NUNCUPATAS, ET AFFECTAS, SED
MORTE AUTHORIS SUI ANNO MDCI. DESERTAS,
JUSSU ET STIPENDIIS FRETUS TRIUM IMPPP.
RUDOLPHI, MATTHIÆ, FERDINANDI,
ANNITENTIBUS HÆREDIBUS BRAHEANIS, EX FUNDAMENTIS OB-
IOANNES KEPLERUS,
Cum Privilegiis, IMP. & Regum Rerúmq; publ. vivo TYCHONI ejúsq; Hæredibus,
& speciali Imperatorio, ipsi KEPLERO concesso, ad Annos XXX.
ANNO M. DC. XXVII.

$$\frac{T^2}{a^3} = \frac{4\pi^2}{G(M+m)}$$

Kepler's laws
fit Brahe's data
Why?

$$\vec{F} = m\,\vec{a}$$

True epistemic
knowledge!

- *Real knowledge is generated by force of reason!*
- *Data-Science is just an interim stopgap measure*

Michael Ortiz
EC25 10/25/22

Concluding remarks

Thank you!