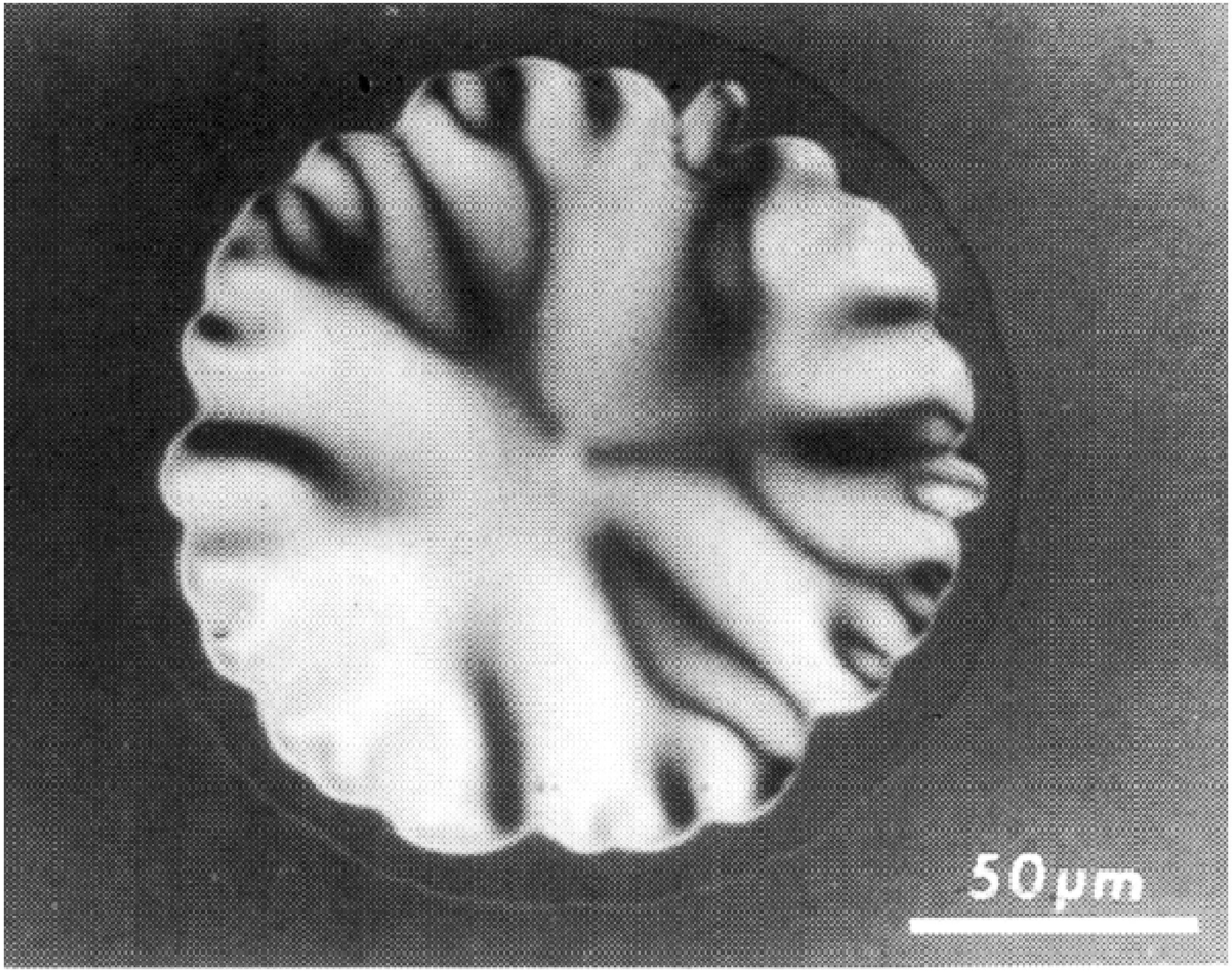
Delamination of Compressed Thin Films

Michael Ortiz
Caltech

Material Interfaces
and Geometrically Based Motions
IPAM/UCLA, April 26, 2001

Blister Morphologies



Blister in SiC film on Si substrate (Argon et al., 1989)

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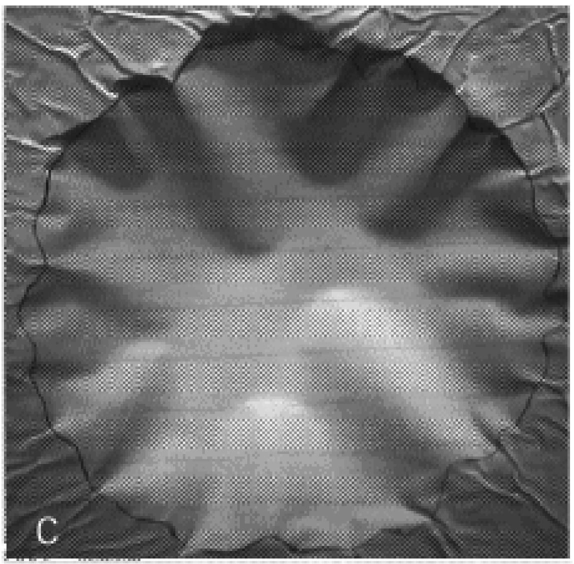
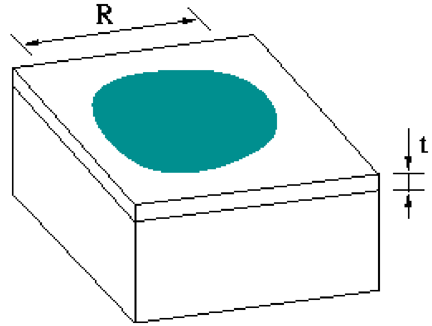
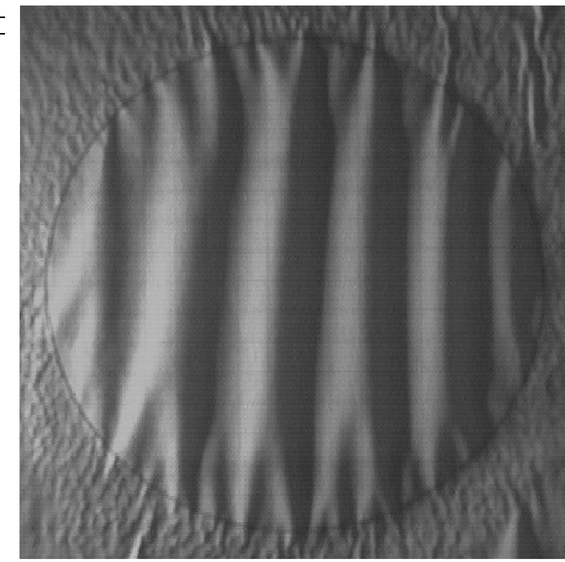
Folding Pattern – Anisotropy







Isotropic compression

Anisotropic compression

(Cuitino and Gioia, unpublished)

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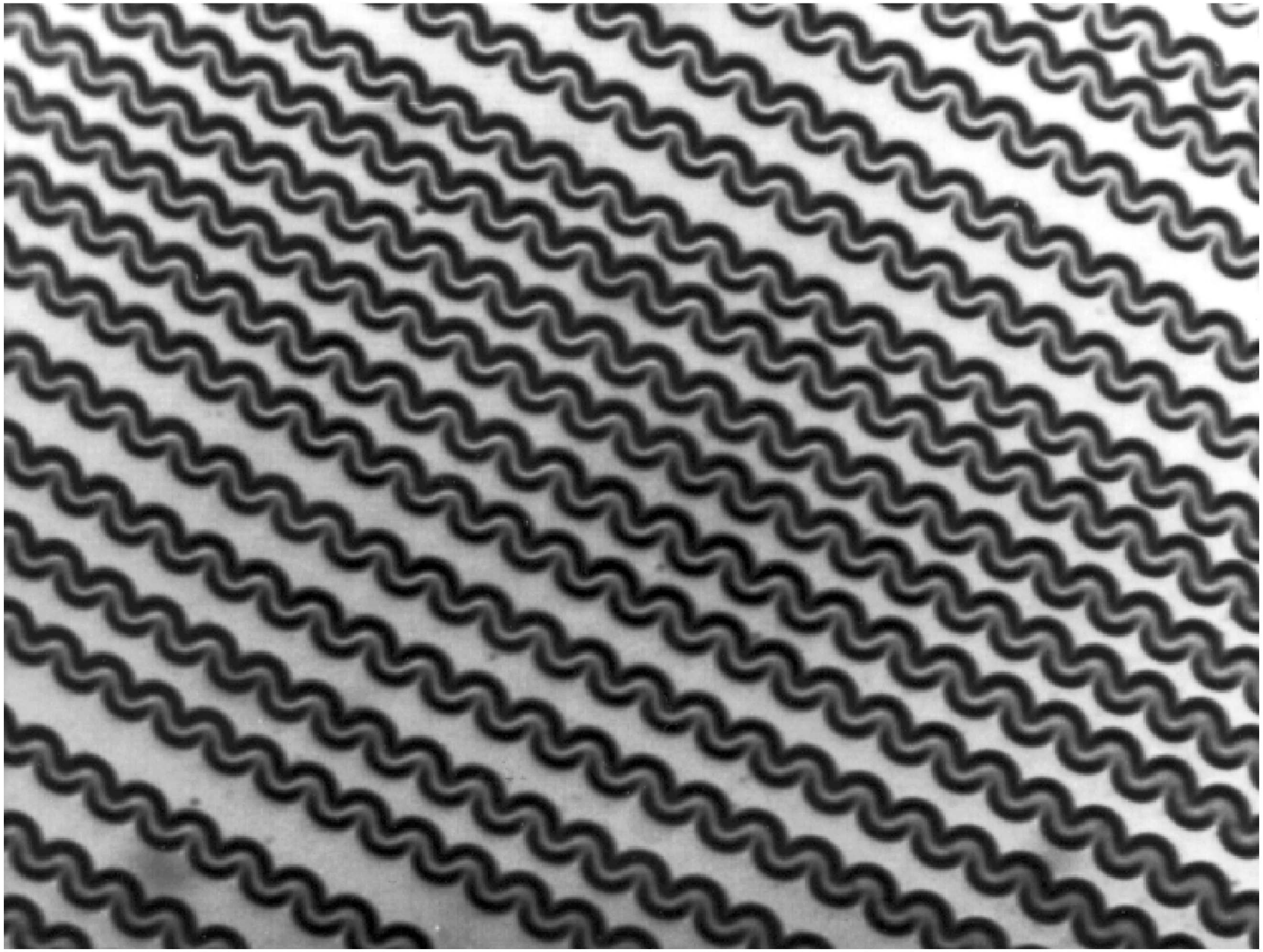
Blister Morphology



Parallel array of telephone-cord blisters (Bastawros and Kim, 1994).

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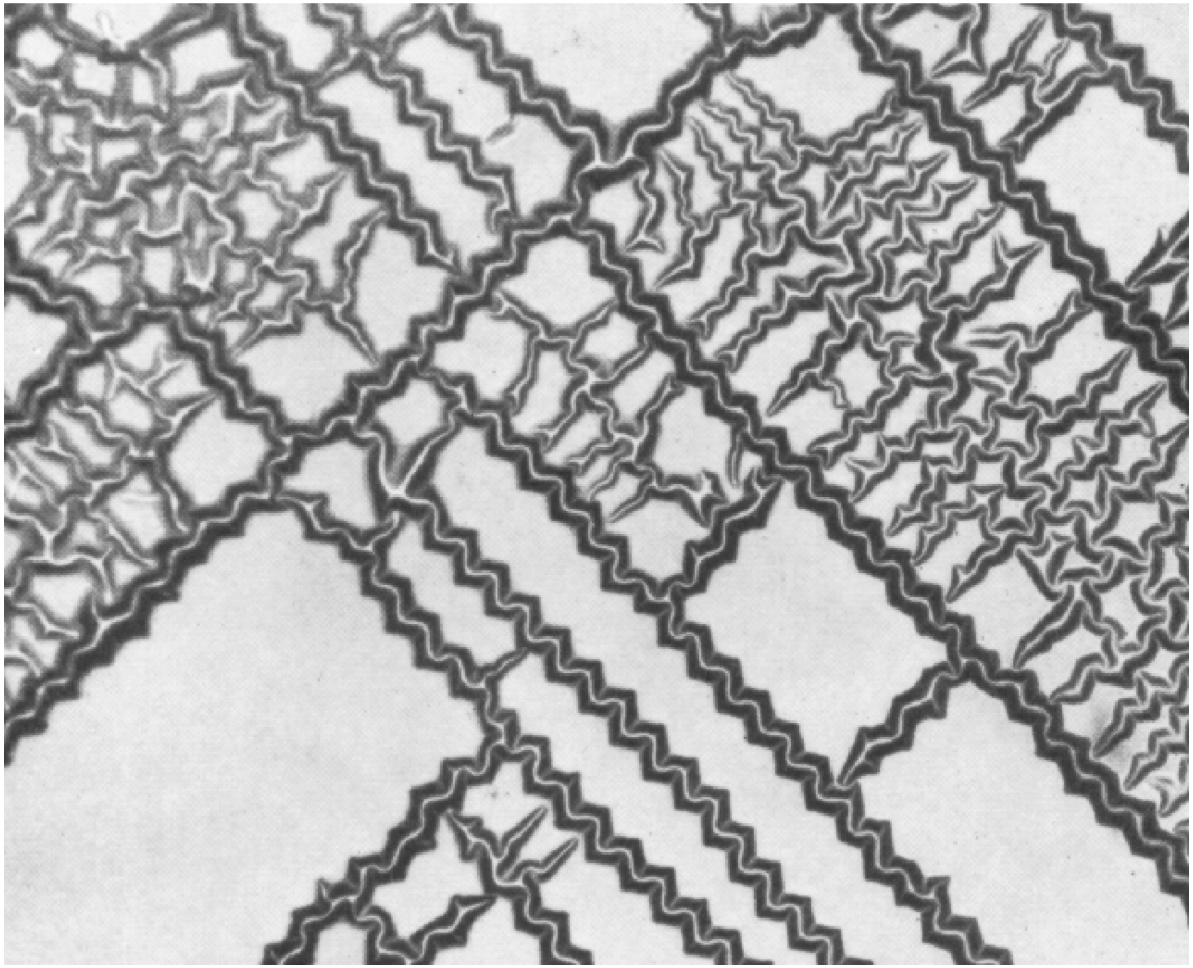
Blister Morphologies



Regular web of telephone cords in a 75--25~Fe-Ni film grown by epitaxial evaporation on rock salt. (Yelon and Voegeli, 1964).

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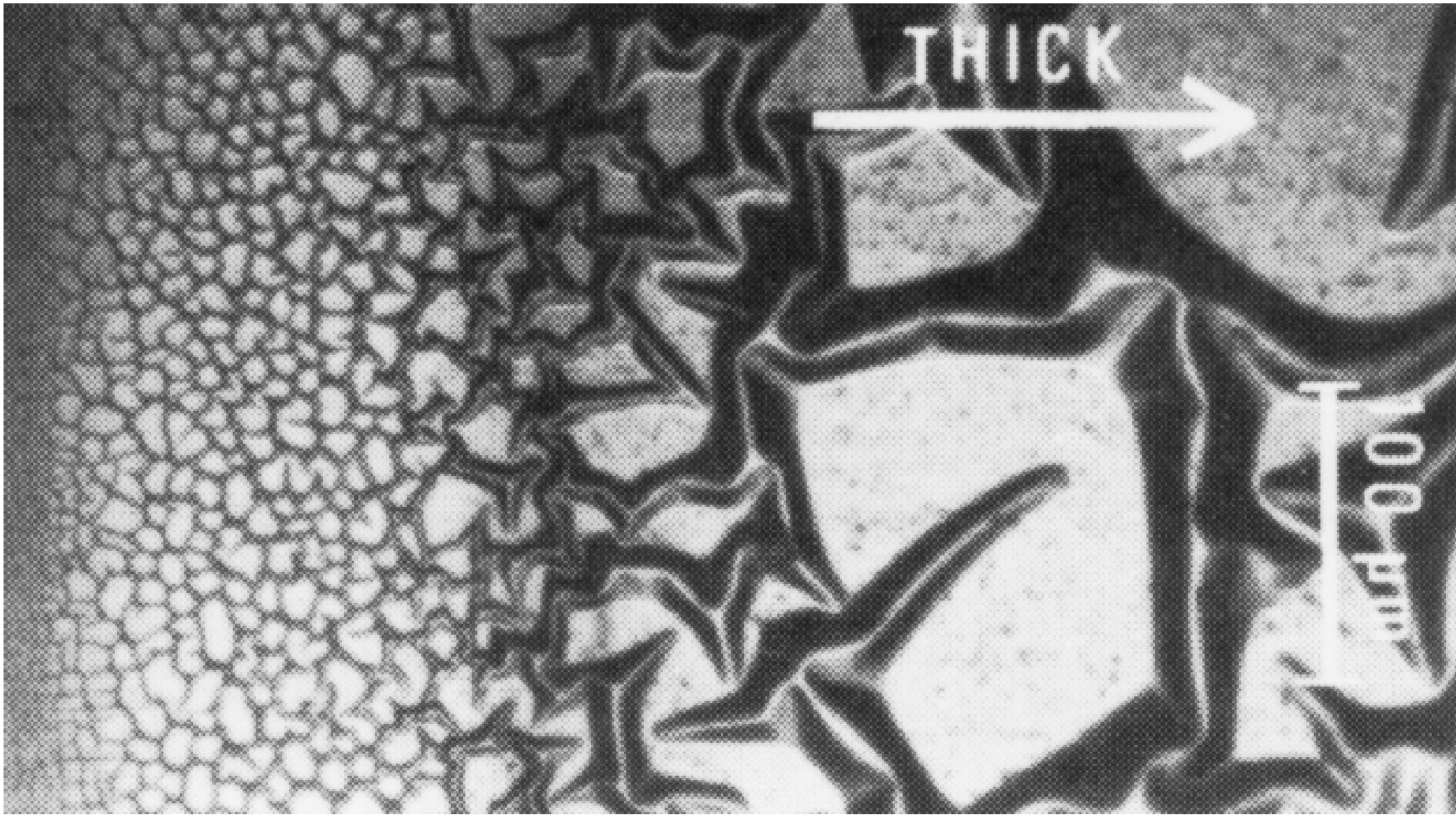
Blister Morphologies



Web of blisters in a carbon film with a thickness gradient
(Matuda et al., 1981)

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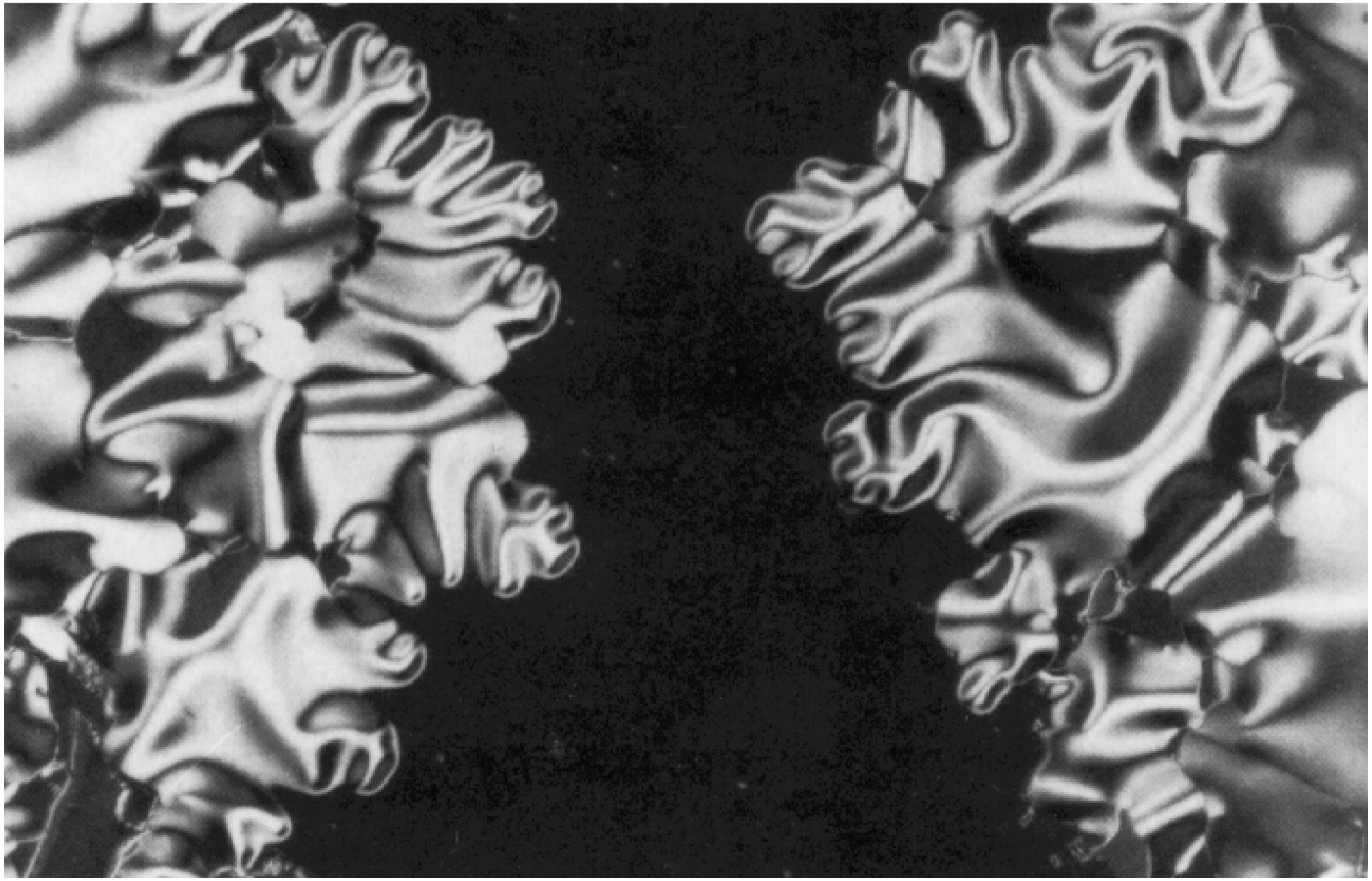
Blister Morphologies



Buckling fronts in a 0.6 micron diamond-like carbon film advancing over Si substrate (Seth et al., 1992)

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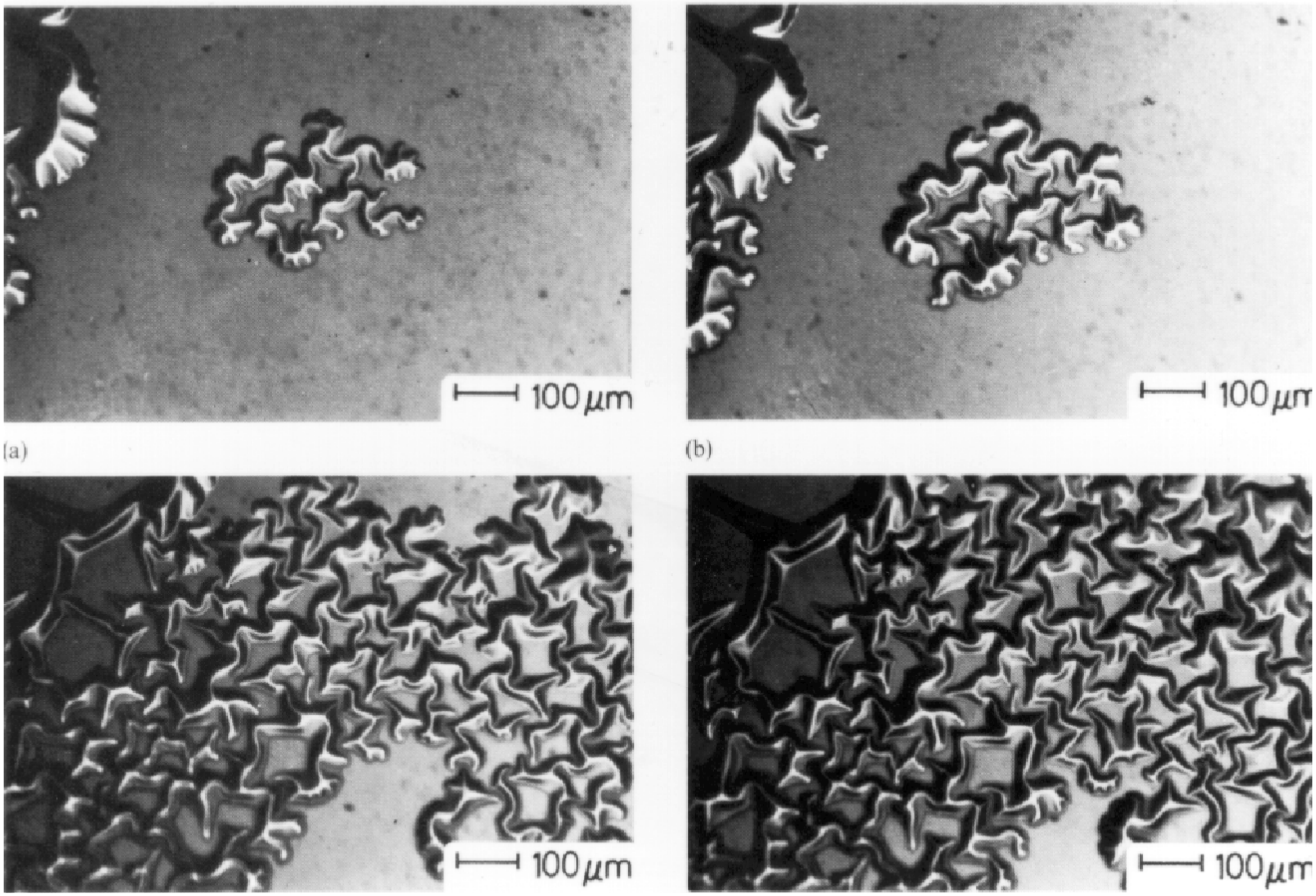
Blister Morphologies



Web of blisters in an 800~\AA~carbon film on a glass substrate (Kinbara et al., 1981). a) 2~h; b) 4~h; c) 10~h; and d) 12~h.

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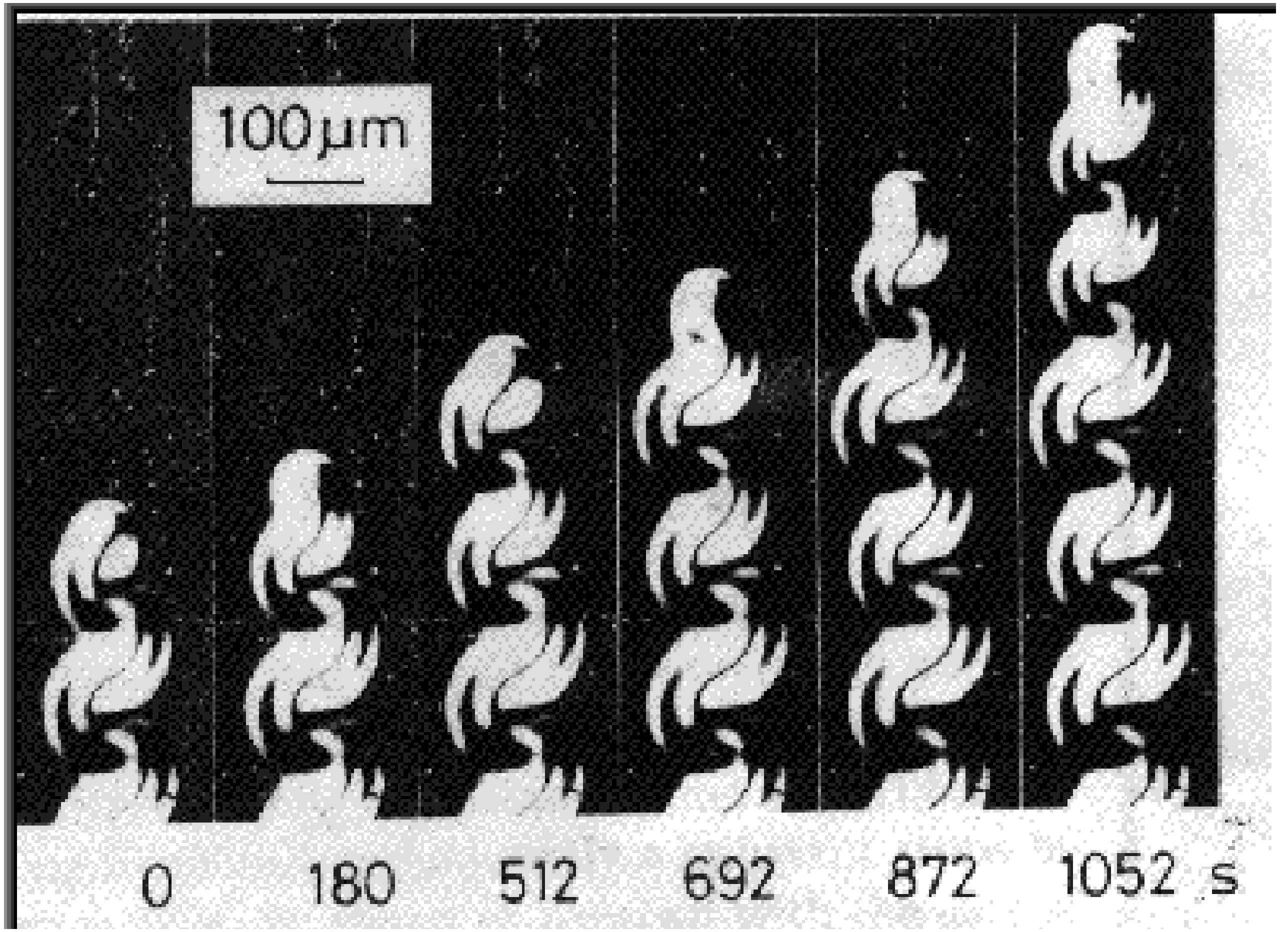
Blister Morphologies



Growth of a telephone-cord blister in a Mo/glass system by delamination at the tip (Ogawa et al., 1986) .

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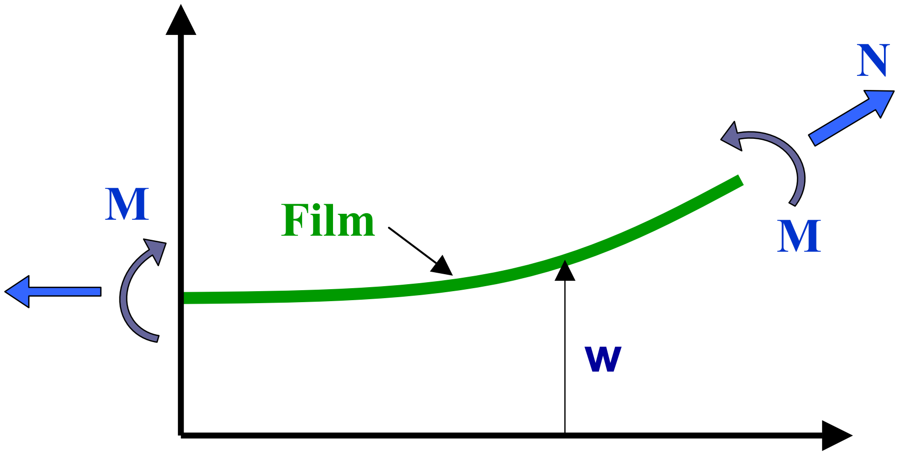
Questions

- What are the *preferred* folding patterns of blisters?
- What are the *preferred* shapes of blisters?
- What are the growth modes, rates, of blisters?

– Ortiz, M. and Gioia, G. ``The Morphology and Folding Patterns Of Buckling Driven Thin-Film Blisters," *J. Mech. Phys. Solids,* **42** (1994) 531-559.
– Gioia, G. and Ortiz, M., ``Delamination of Compressed Thin Films," *Adv. Appl. Mech.,* **33** (1997) 119-192.
– Cuitino, A.M., Gioia, G., DeSimone, A. and Ortiz, M. ``Folding Energetics in Thin-Film Diaphragms," TAM Report #939, UIUC, April, 2000.
– Cirak, F., Cuitino, A.M., Gioia, G., Ortiz, M., Smart, M. ``A Numerical Study of Anisotropically Compressed Thin-Film Diaphragms," ASME/WAM, Orlando, FL, November, 2000.

Von Karman Theory of plate bending



- Energy of the film:

$$\Phi[w] = \Phi^{\mathrm{m}}[w] + \Phi^{\mathrm{b}}[w]$$

- Membrane energy:

$$\Phi^{\mathrm{m}}[w] = \int_\Omega \frac{C}{8}(|\nabla w|^2 - k^2)^2 d^2x$$

$$k = \sqrt{2(1+\nu)\varepsilon^*}$$ (Preferred slope)

- Bending energy: $\Phi^{\mathrm{b}}[w] =$

$$\int_\Omega \frac{D}{2}[(1-\nu) w,_{\alpha\beta} w,_{\alpha\beta} + \nu (w,_{\gamma\gamma})^2] d^2x$$

- Problem: $\boxed{\inf \Phi[w]}$

$$w = 0, \ \ w,_n = 0 \text{ on } \Gamma$$

- Kinematics

$$\varepsilon_{\alpha\beta} = w,_\alpha w,_\beta /2 - \varepsilon^*_{\alpha\beta}$$

$$\chi_{\alpha\beta} = w,_{\alpha\beta}$$

Eigenstrains

- Isotropic compression:

$$\varepsilon^*_{\alpha\beta} = \varepsilon^* \delta_{\alpha\beta}$$

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Thin-film limit

- Membrane and bending energies:

$$\Phi^{\mathrm{m}}[w] = \int_\Omega \frac{C}{8}(|\nabla w|^2 - k^2)^2 d^2x \longrightarrow C = \frac{Eh}{1-\nu^2};$$

$$\Phi^{\mathrm{b}}[w] = \int_\Omega \frac{D}{2}[(1-\nu) w,_{\alpha\beta}\, w,_{\alpha\beta} + \nu (w,_{\gamma\gamma})^2] d^2x \rightarrow D = \frac{Eh^3}{12(1-\nu^2)}$$

- As h $\rightarrow$ 0, bending energy singularly perturbs membrane energy.
- Membrane problem: $\inf \Phi^{\mathrm{m}}[w] \Rightarrow \boxed{|\nabla w| = k}$ eikonal equation

Nonconvex!

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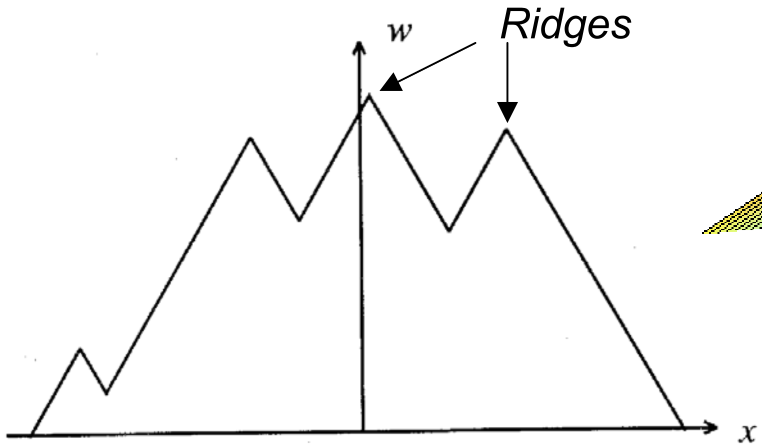
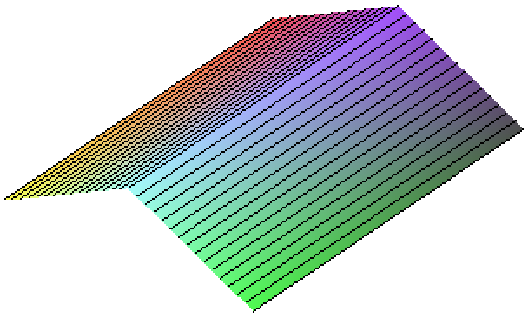
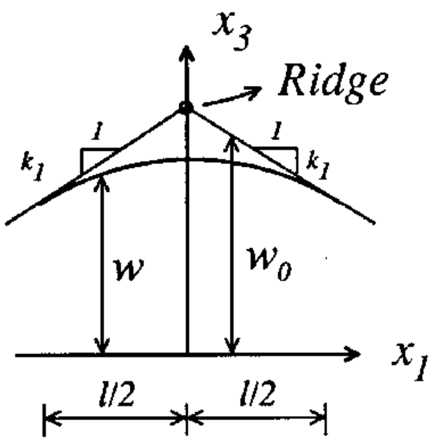
Ridge energy







- Ridge energy per unit length: $\dfrac{\Phi}{L} = \dfrac{2}{3}\sqrt{CD}k_1^3$
- Conjecture: Let $|\nabla w| = k\ a.\ e.\ \text{in}\ \Omega$

$$\Phi_0[w] = \Gamma - \lim_{h\to 0} \Phi[w] = \int_\Omega \frac{1}{3}\sqrt{CD}|w_{,\alpha\beta}\, w_{,\alpha}\, w_{,\beta}\,|d^2x$$

- Energy concentrated on singular set (ridges)
- Jin and Kohn, J Nonlinear Sci, Vol 10 (2000) 355: Sharp bounds support conjecture.

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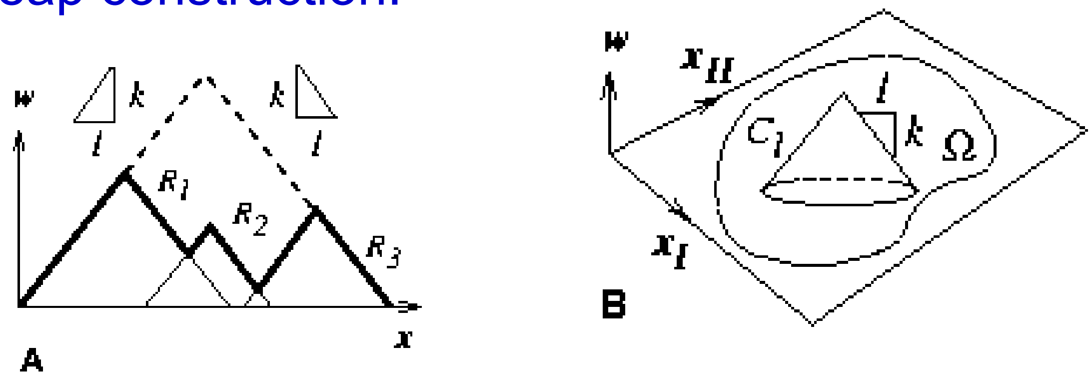
Gamma-Limit problem

- Limiting energy: Let

$$\Phi_0[w] = \Gamma - \lim_{h \to 0} \Phi[w] = \int_\Omega \frac{1}{3}\sqrt{CD}|w_{,\alpha\beta}\, w_{,\alpha}\, w_{,\beta}\,|d^2x$$

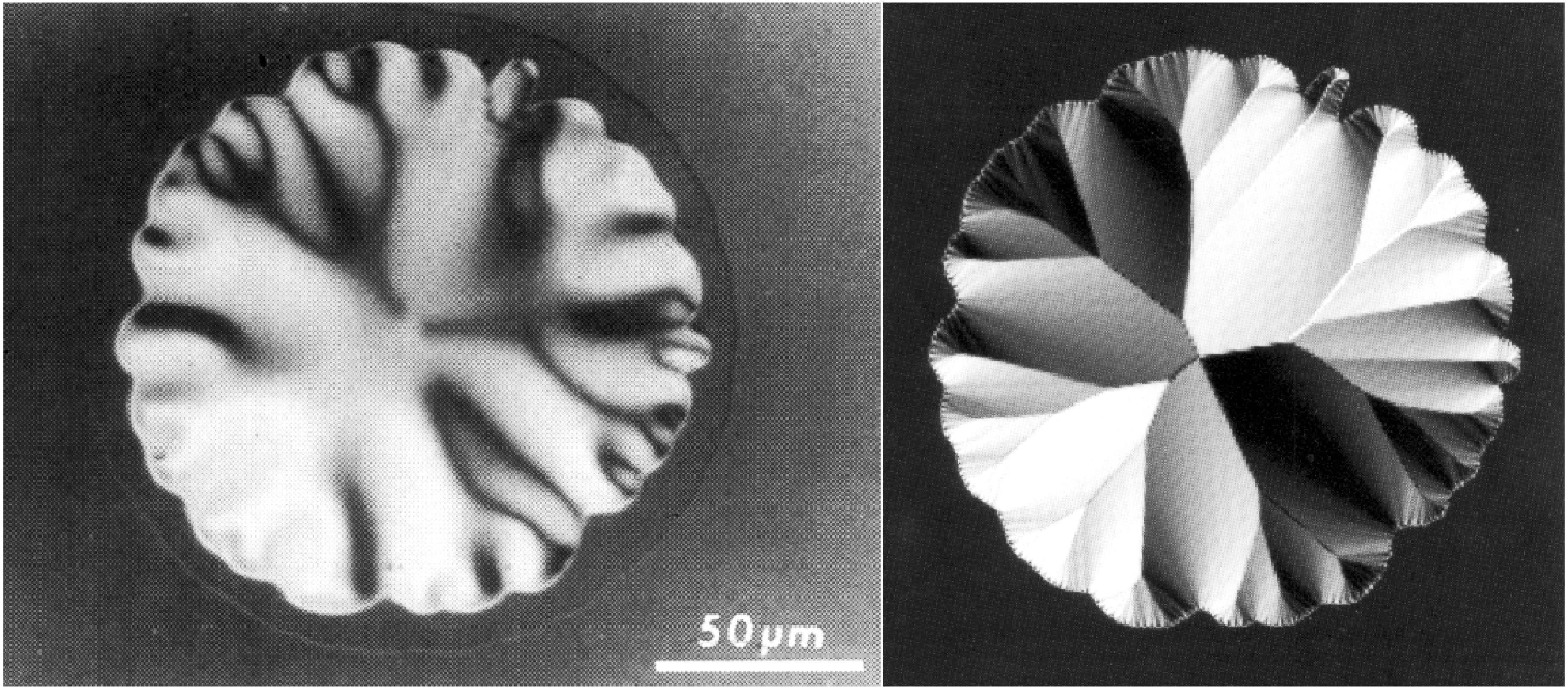
- Limiting problem: $\inf \Phi_0[w], \quad \text{subject to } |\nabla w| = k \ a.\ e. \text{ in } \Omega$

- Conjecture (Ortiz and Gioia, 1994): $\boxed{w(\boldsymbol{x}) = d(\boldsymbol{x}, \Gamma)}$
  Sand-heap construction!



- Jin and Kohn, J Nonlinear Sci, Vol 10 (2000) 355: Other constructions may deliver same energy

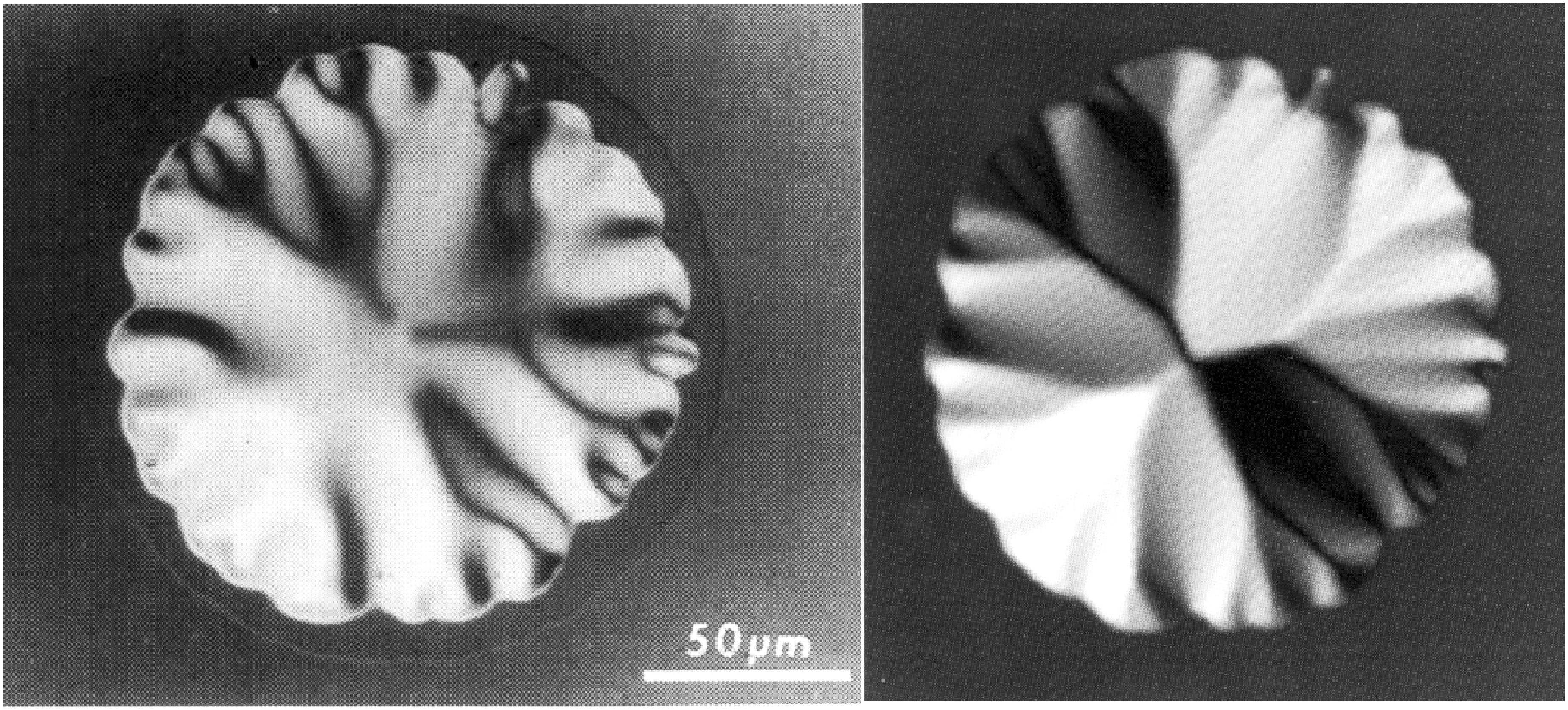
Membrane Solution



(a) (b)

a) Blister in SiC/Si (Argon et al., 1989)
b) Membrane solution (Ortiz and Gioia, 1994)

Bending Solution



(a) (b)

a) Blister in SiC/Si (Argon et al., 1989)
b) Membrane solution (Ortiz and Gioia, 1994)

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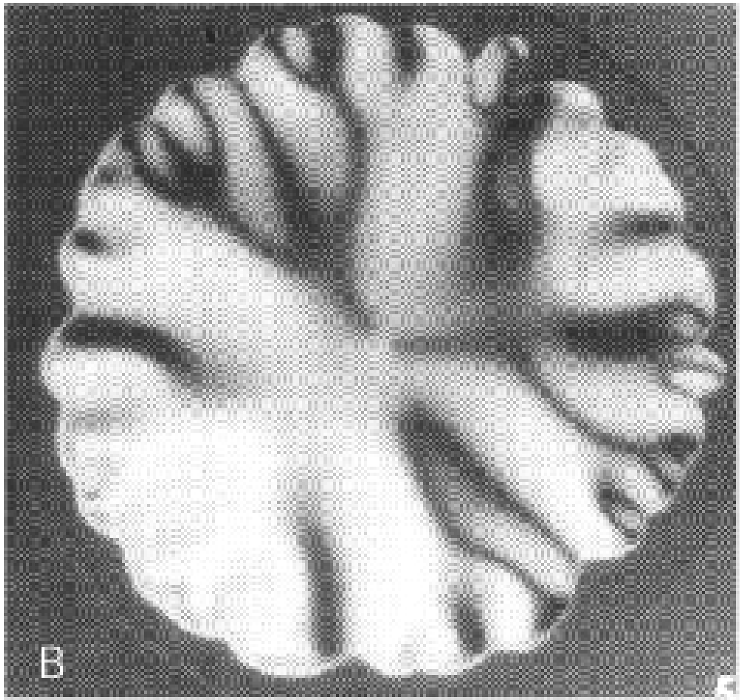
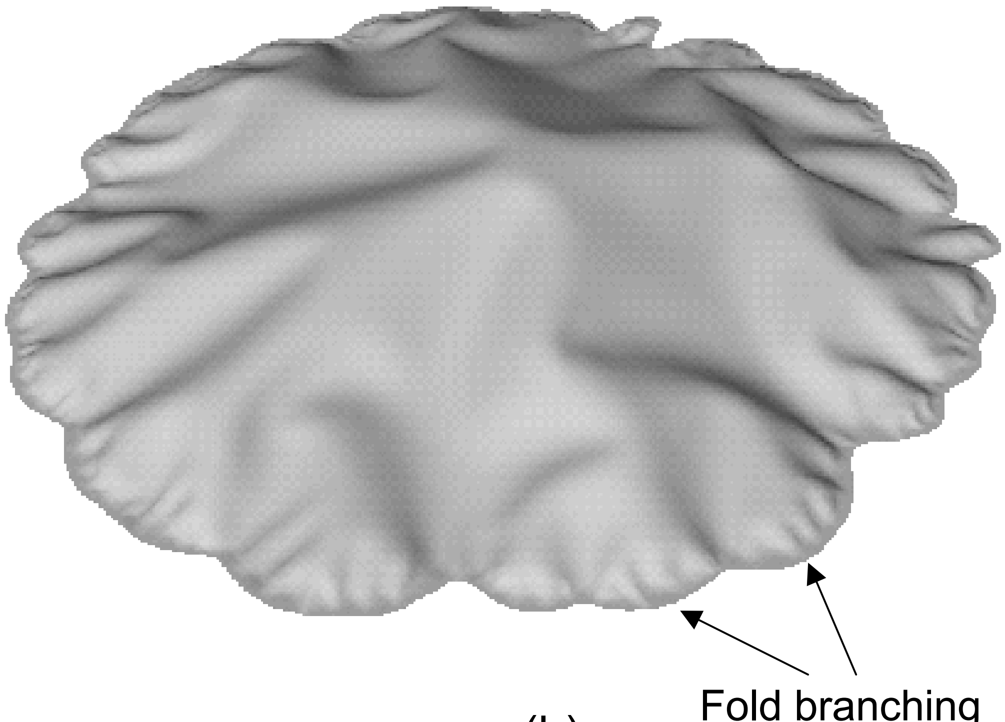
Finite-deformation shell solution



(a)



(b)

a) Blister in SiC/Si (Argon et al., 1989)
b) Finite-deformation shell solution
(Cirak, Ortiz, Gioia, Smart and Cuitino, 2000)

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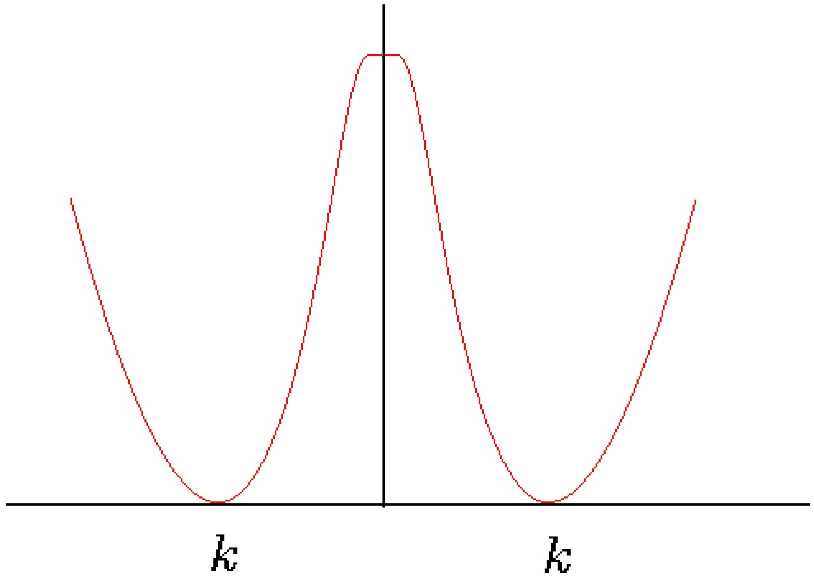
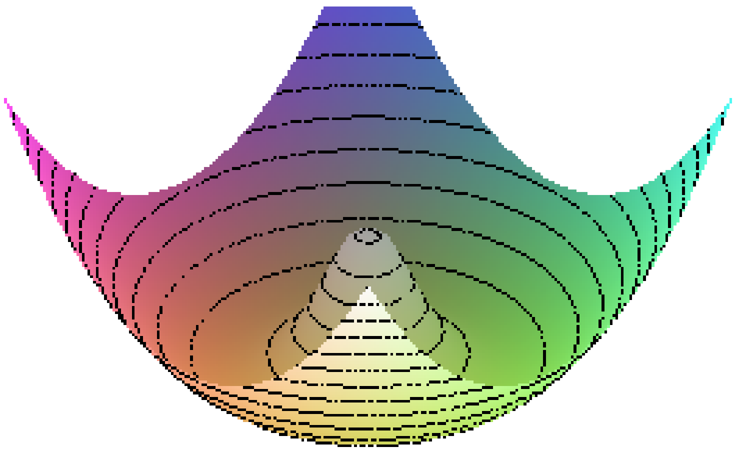
Membrane minimizers - Isotropic

- Isotropic compression: $\epsilon_I^* = \epsilon_{II}^*$

$$|\nabla w| = k, \text{ where } k \equiv \sqrt{2(\epsilon_I^* + \nu \epsilon_{II}^*)}$$



$$\textit{compressive regime}, \ \epsilon_I^* > 0$$



Membrane energy density

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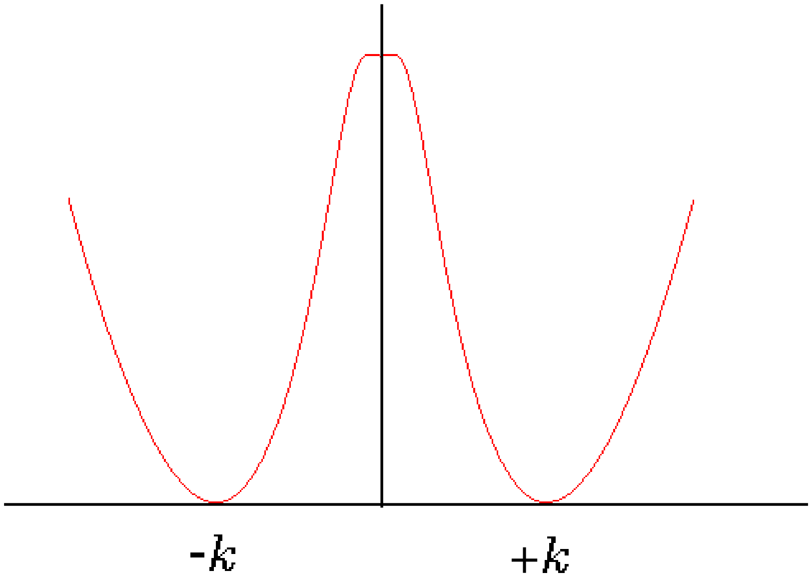
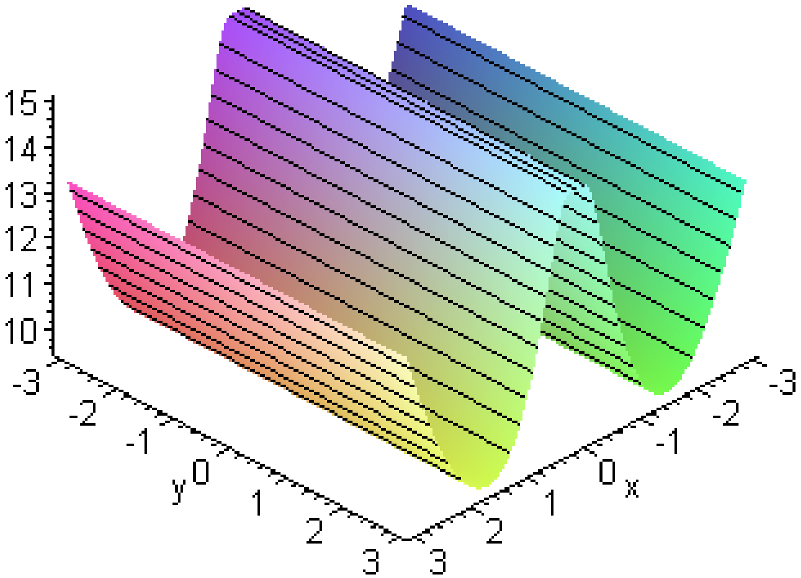
Membrane minimizers - Anisotropic

•Anisotropic compression: $\epsilon_I^* \neq \epsilon_{II}^*$

$$\nabla w \longrightarrow (\pm k, 0) \qquad \textit{compressive regime}, \ \epsilon_I^* + \nu \epsilon_{II}^* > 0$$





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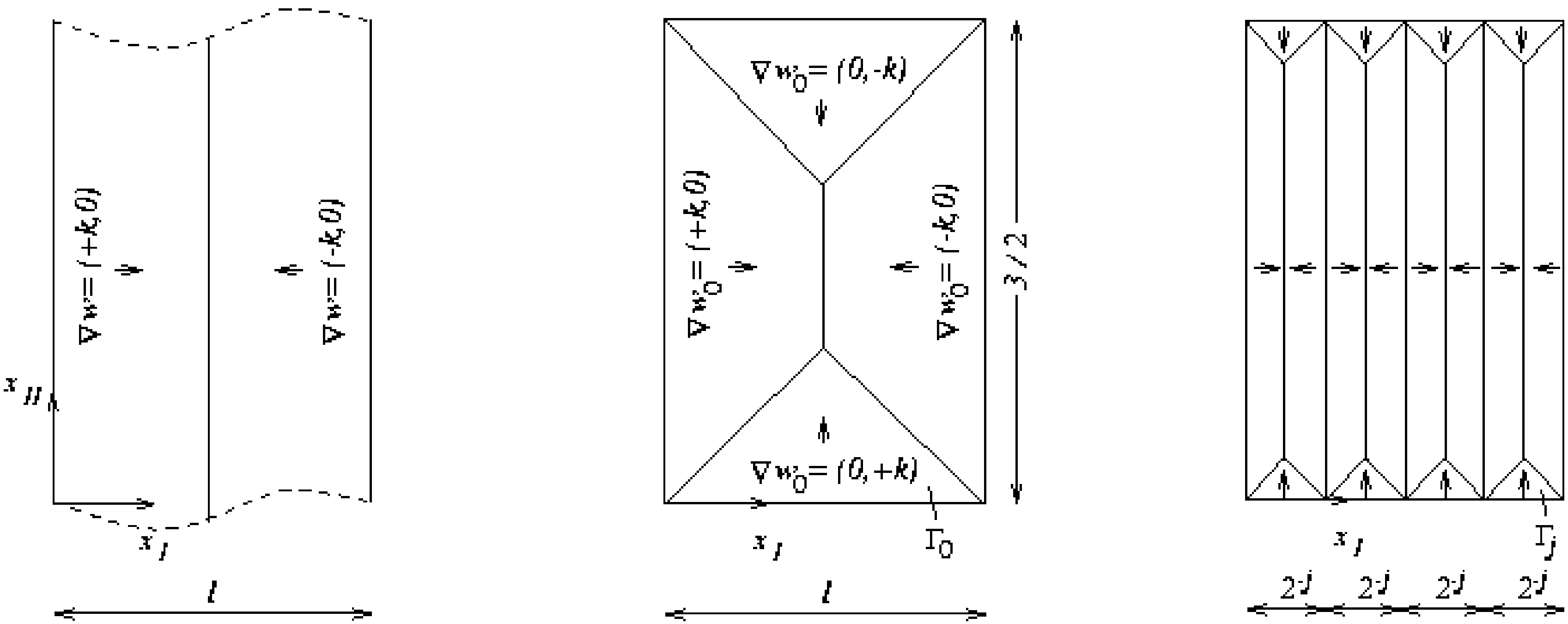
Membrane construction - Anisotropic

Anisotropic compression: $\epsilon^*_I \neq \epsilon^*_{II}$



Φ^m can be driven arbitrarily close to its infimum by allowing the diaphragm to develop fine folding

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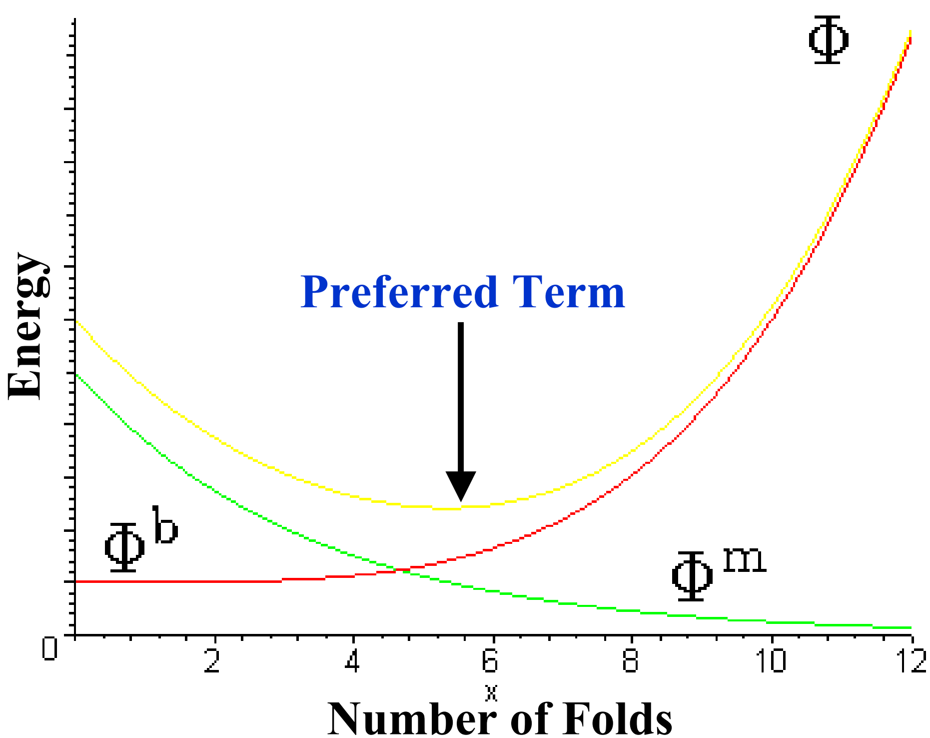
Gamma-limit construction - Anisotropic

ANISOTROPIC $\epsilon_I^* \neq \epsilon_{II}^*$



Implications

- No folding exist which ca accommodate the boundary conditions and minimize Φ^m
- The infimum of Φ^m can be approached by allowing the folding to increase without limit (minimizing sequence)
- Bending selects a preferred folding

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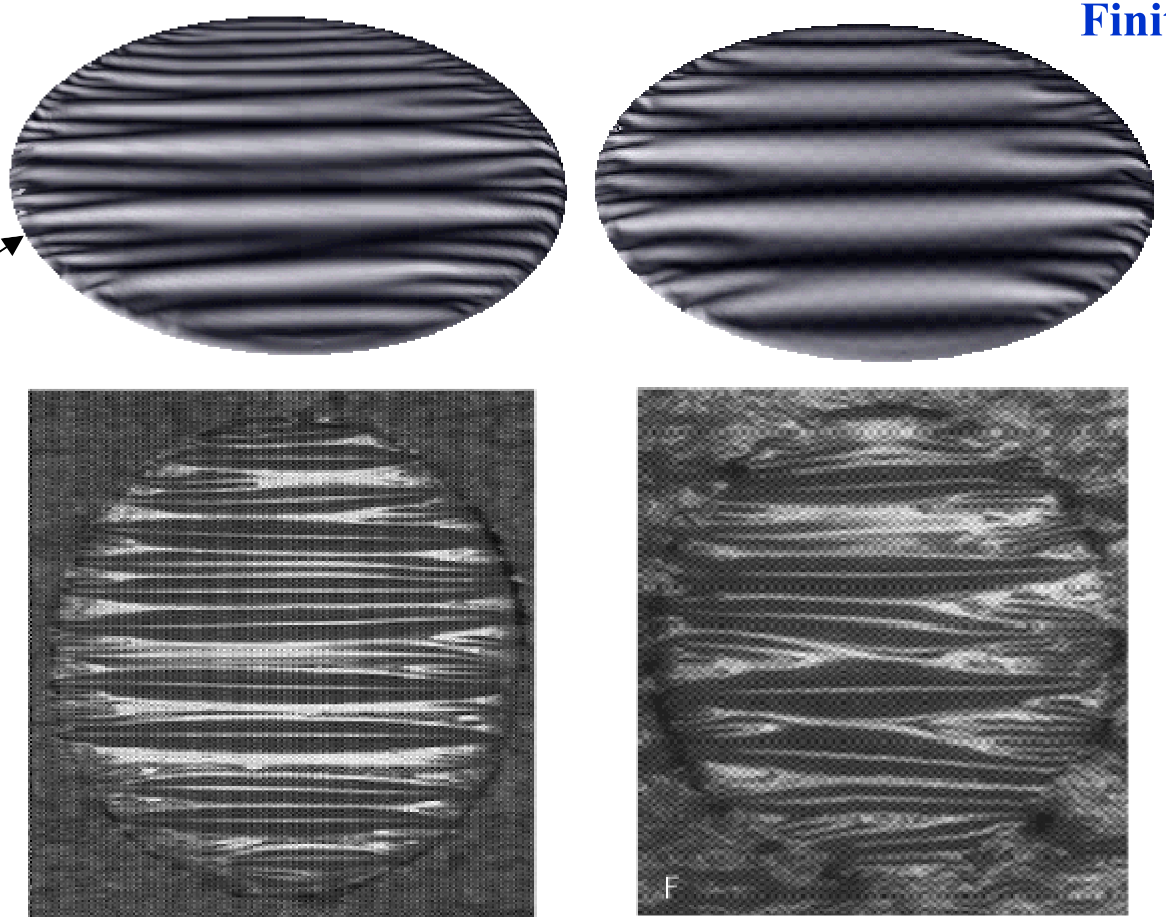
Experimental validation

Finite elements

Fold branching



Experimental

Belgacem, Conti, DeSimone and Muller, J. Nonlinear Sci, 10 (2000) 661:
Fold branching delivers optimal energy scaling.

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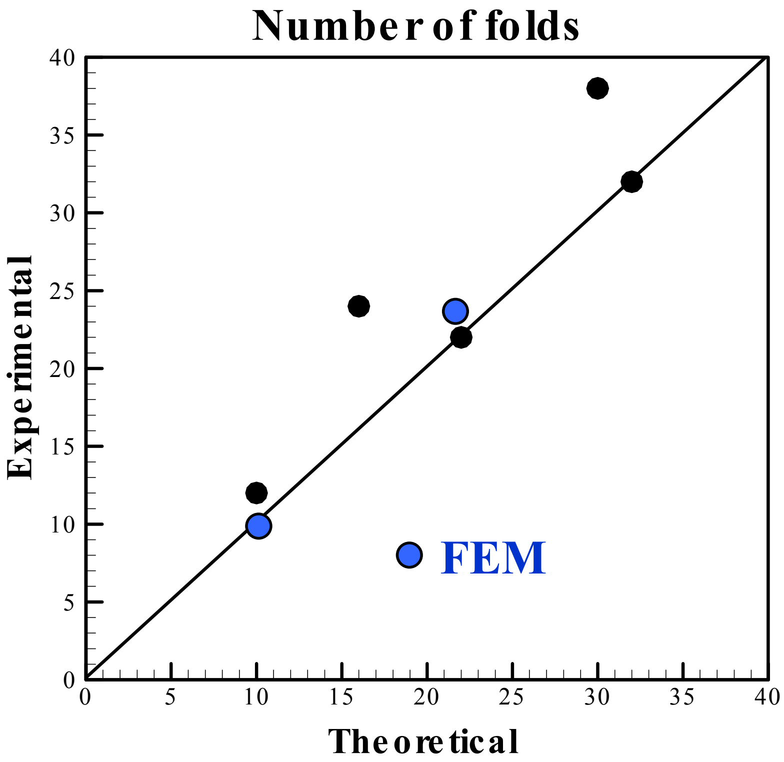
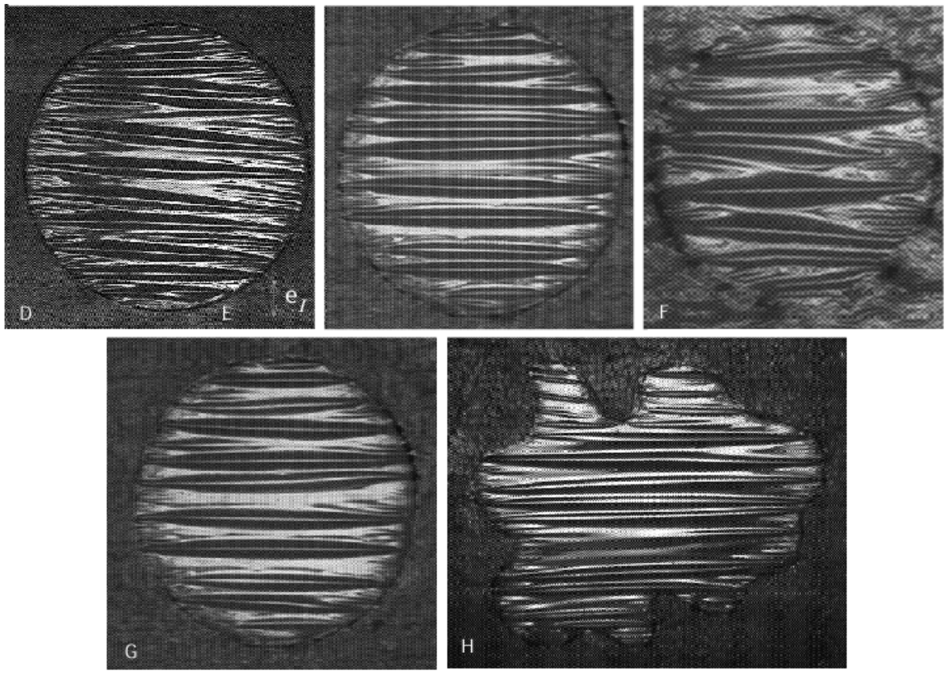
Experimental Validation

Anisotropic compression



Number of folds: $N \sim d/h\,(\varepsilon^*)^{1/2}$



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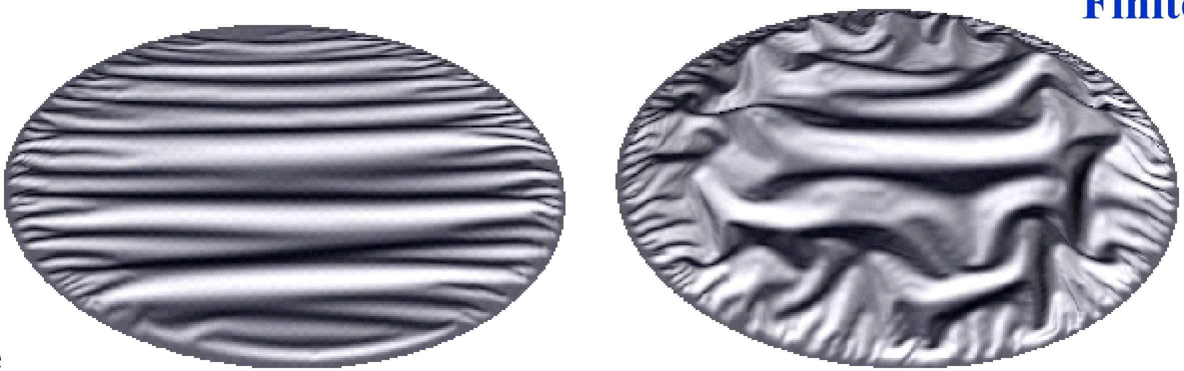
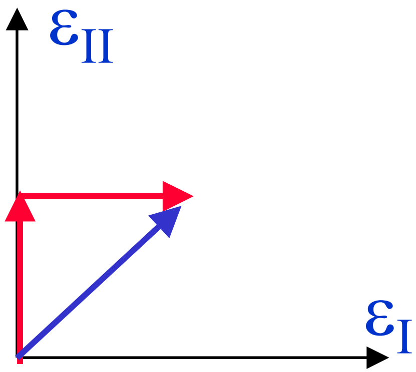
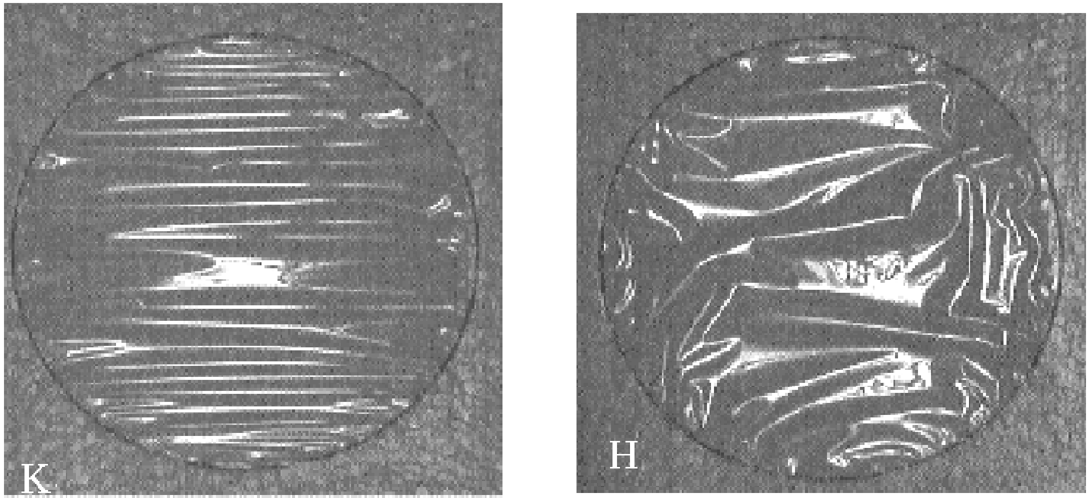
Path-dependence - Metastability

Finite elements



Path Dependence





Experimental

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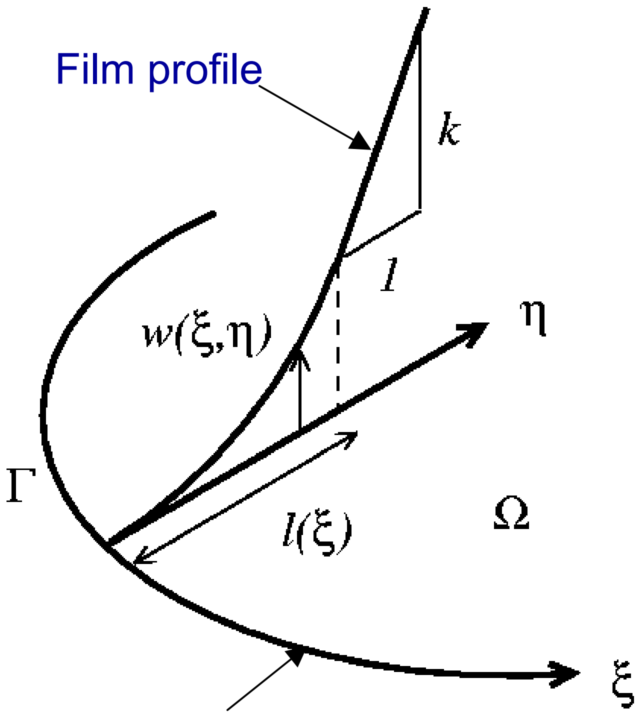
Delamination kinetics



Boundary layer geometry

- Configurational (driving) force for delamination:

$$G = \frac{D}{2}\chi^2$$

- Bending strain:

$$\chi = w_{,\eta\eta}$$

- Kinetic growth law:

$$v = \begin{cases} (G - G_c)/B & \text{if } G > G_c \\ 0 & \text{otherwise} \end{cases}$$

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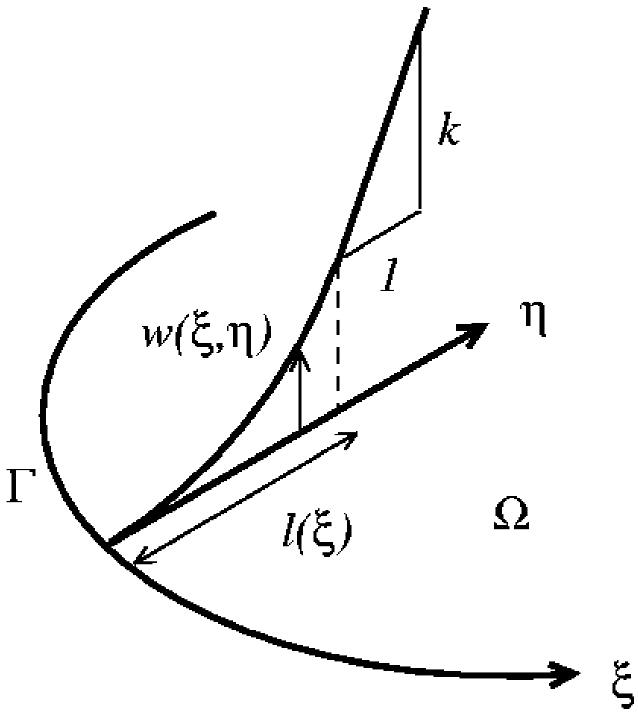
Boundary-layer analysis



- Boundary-layer energy:

$$\Phi[\chi] = \int_0^L T(\kappa, \kappa'; \chi, \chi', \chi'')\, d\xi$$

- Boundary-layer equilibrium

$$f(\kappa, \kappa', \kappa'', \kappa'''; \chi, \chi', \chi'', \chi''', \chi^{iv}) \equiv$$

$$\frac{\partial T}{\partial \chi} - \frac{d}{d\xi}\frac{\partial T}{\partial \chi'} + \frac{d^2}{d\xi^2}\frac{\partial T}{\partial \chi''} = 0$$

- Everywhere critical boundary:

$$\boxed{G = G_c} \quad \text{on } \Gamma$$

$$f(\kappa, \kappa', \kappa'', \kappa'') = 0$$

Equation for boundary curvature as a function of arc length (shape)

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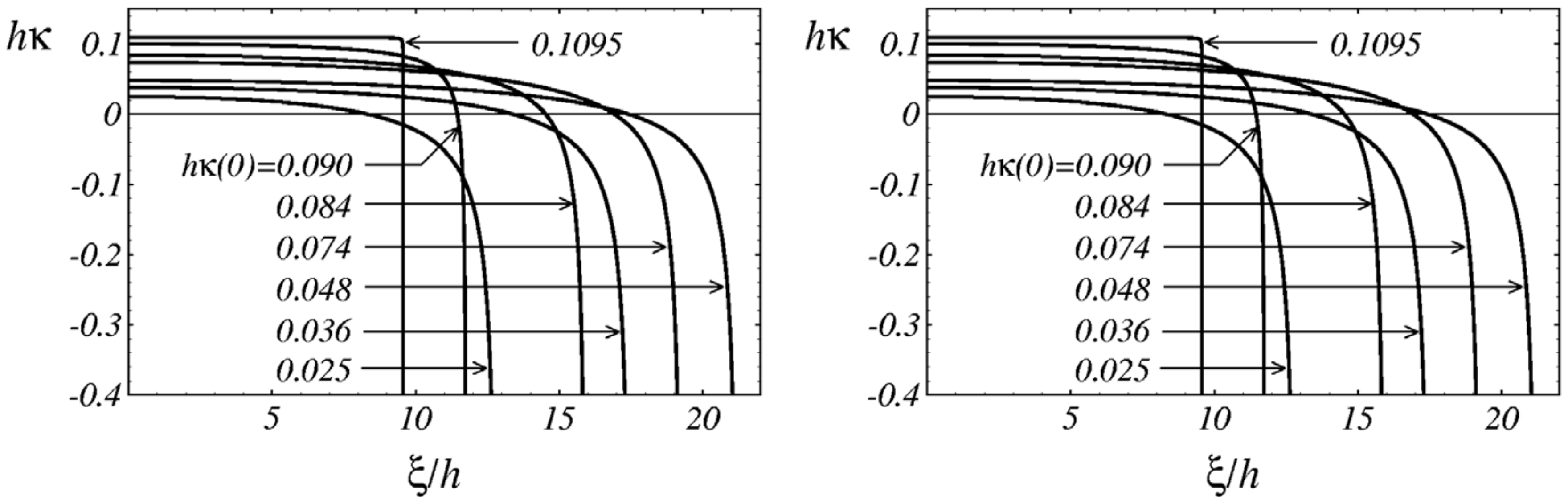
Everywhere-critical boundary shapes



Boundary curvature and tangent angle as a function of arc length

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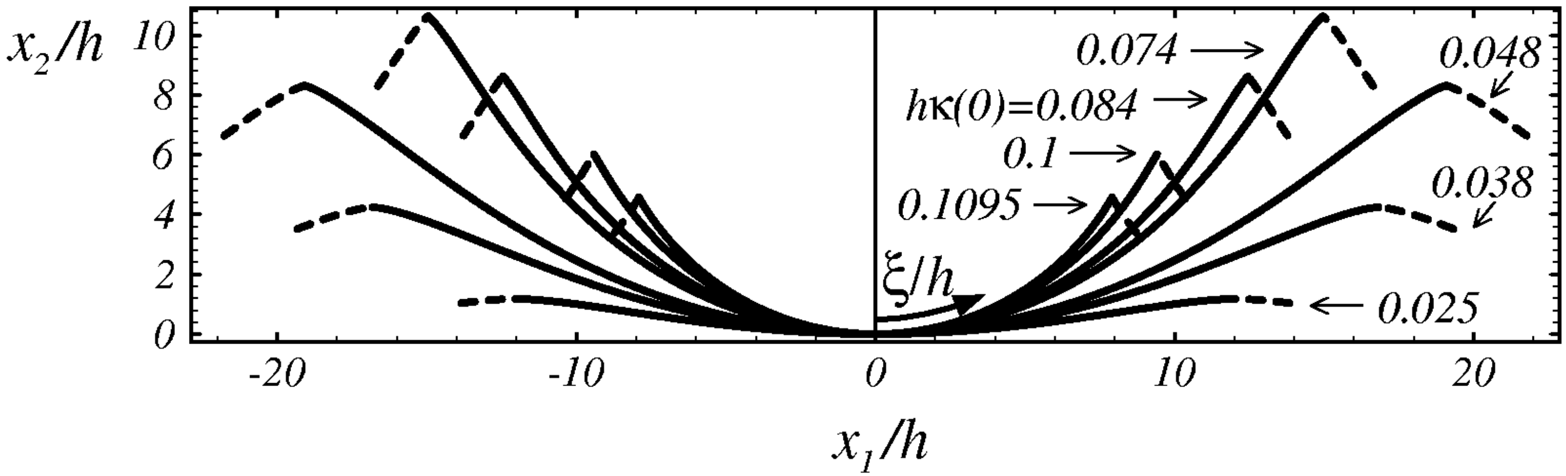
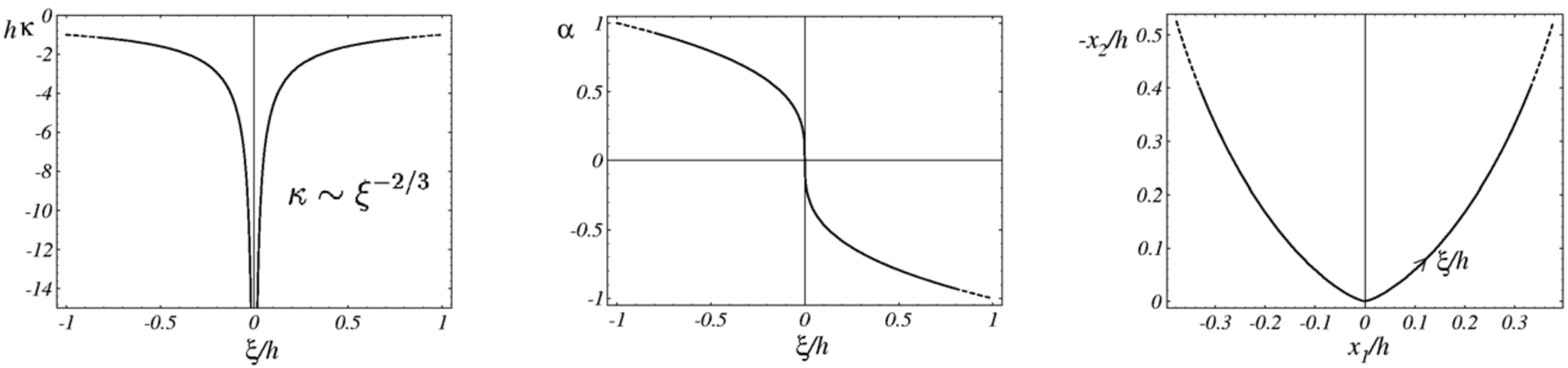
Everywhere-critical boundary shapes



Geometry of everywhere-critical boundary



Cusp geometry

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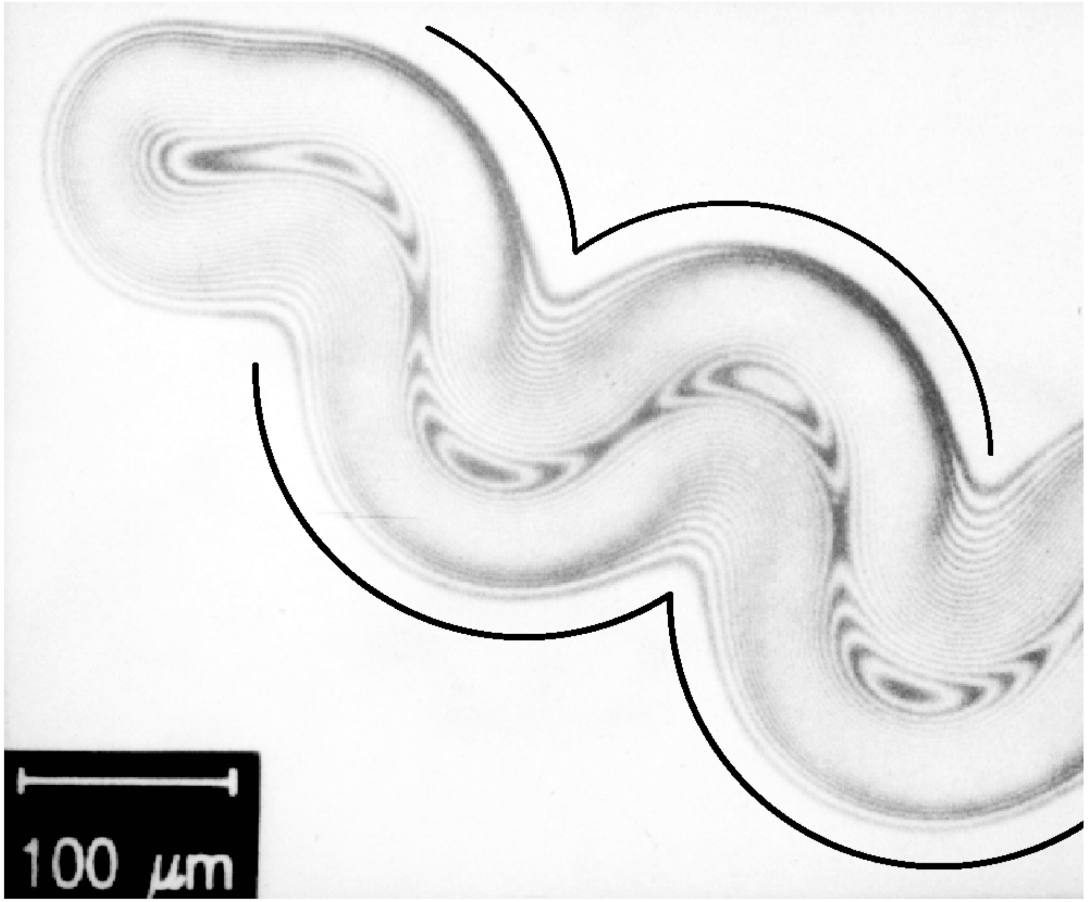
Experimental validation



Comparison of telephone cord in a Si/SiO2~glass system reported by Thouless (1993) and boundary shape predicted by theory (Gioia and Ortiz, 1997)

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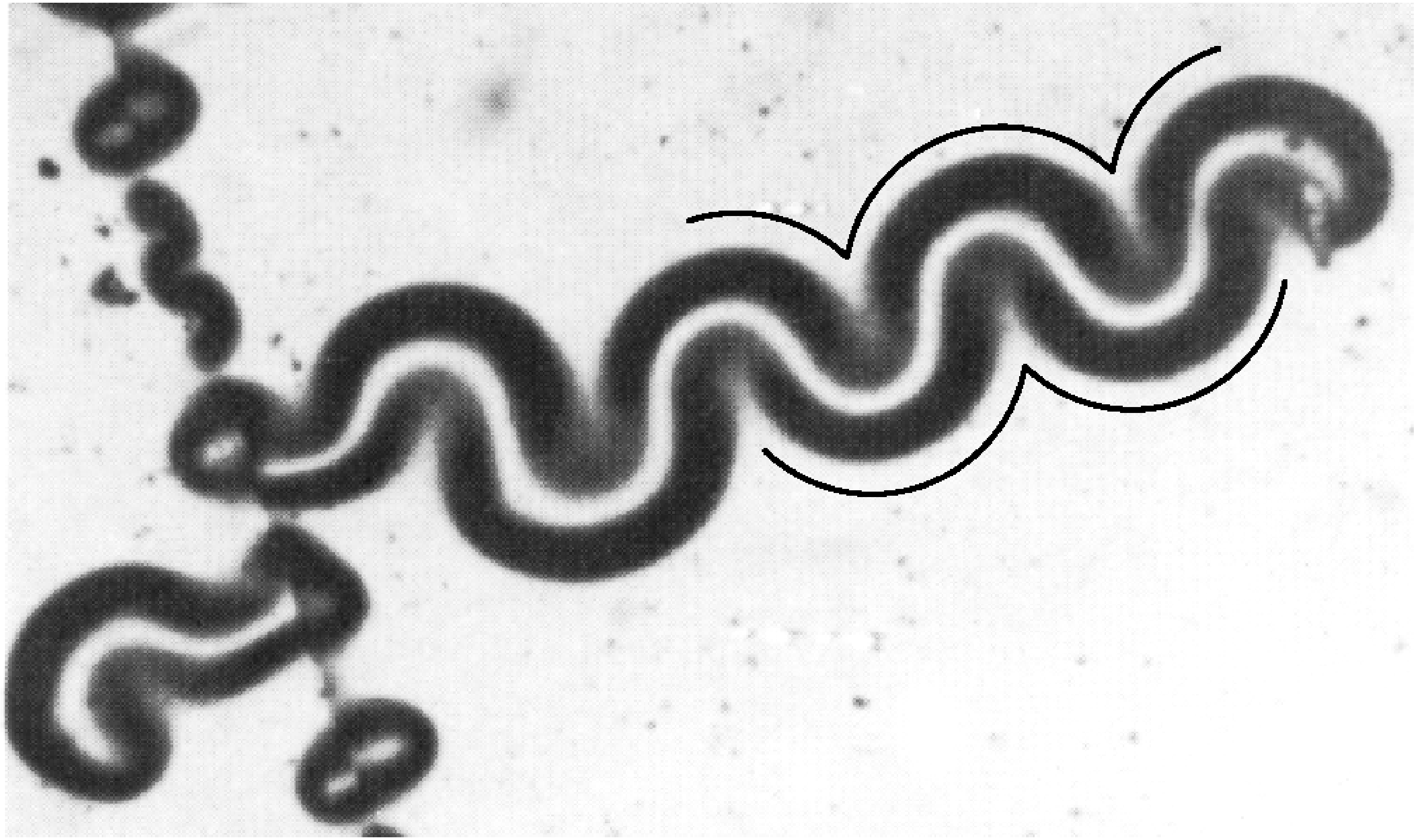
Experimental validation



Comparison of telephone cord in a diamond-like carbon/glass system reported by Nir (1984) and boundary shape predicted by theory (Gioia and Ortiz, 1997).

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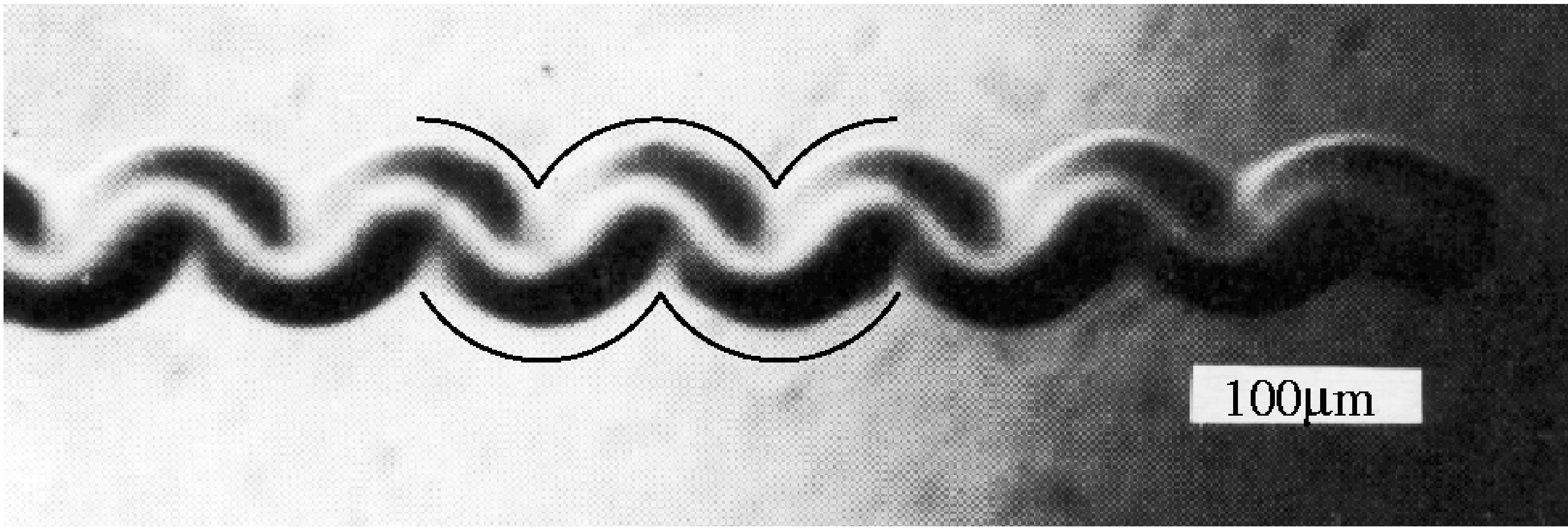
Experimental validation



Comparison of telephone cord in a compositionally modulated Fe/Ni film reported by Yu, Kim and Sanday (1991) and boundary shape predicted by theory (Gioia and Ortiz, 1997)

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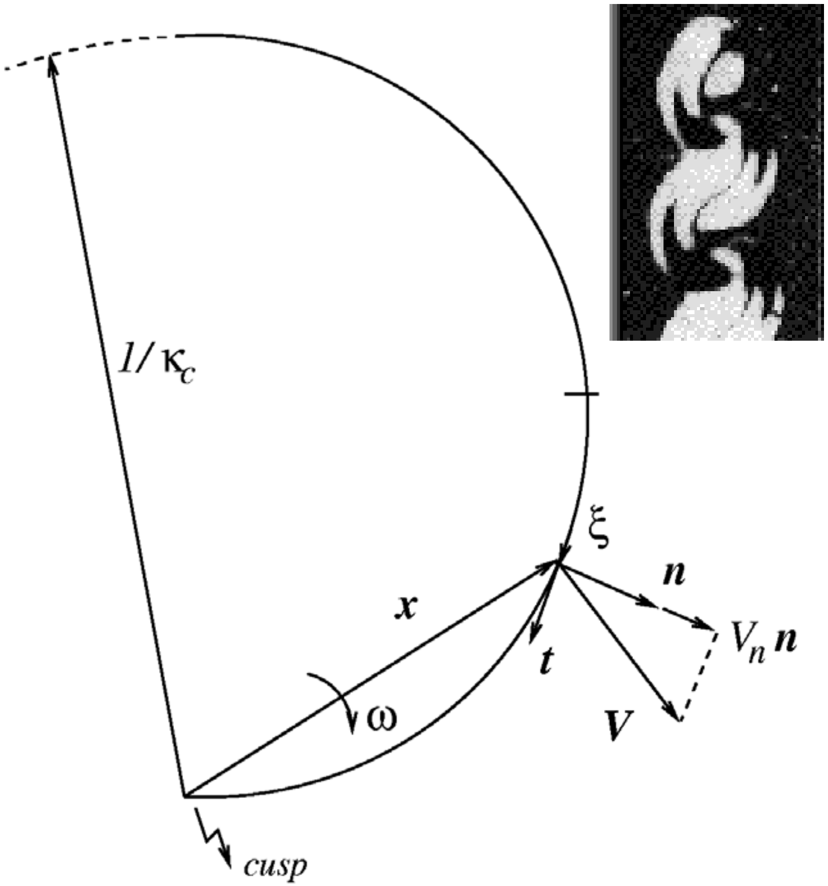
Telephone-cord growth



- Kinetic growth law:

$$v = \begin{cases} (G - G_c)/B & \text{if } G > G_c \\ 0 & \text{otherwise} \end{cases}$$

- Governing equation:

$$2\kappa'' + \kappa(\kappa^2 - \kappa_c^2) = 0$$

- General solution:

$$\xi = \int_{\kappa_0}^{\kappa} \frac{2d\kappa}{\sqrt{(\kappa_0^2 - \kappa_c^2)^2 - (\kappa^2 - \kappa_c^2)^2}}$$

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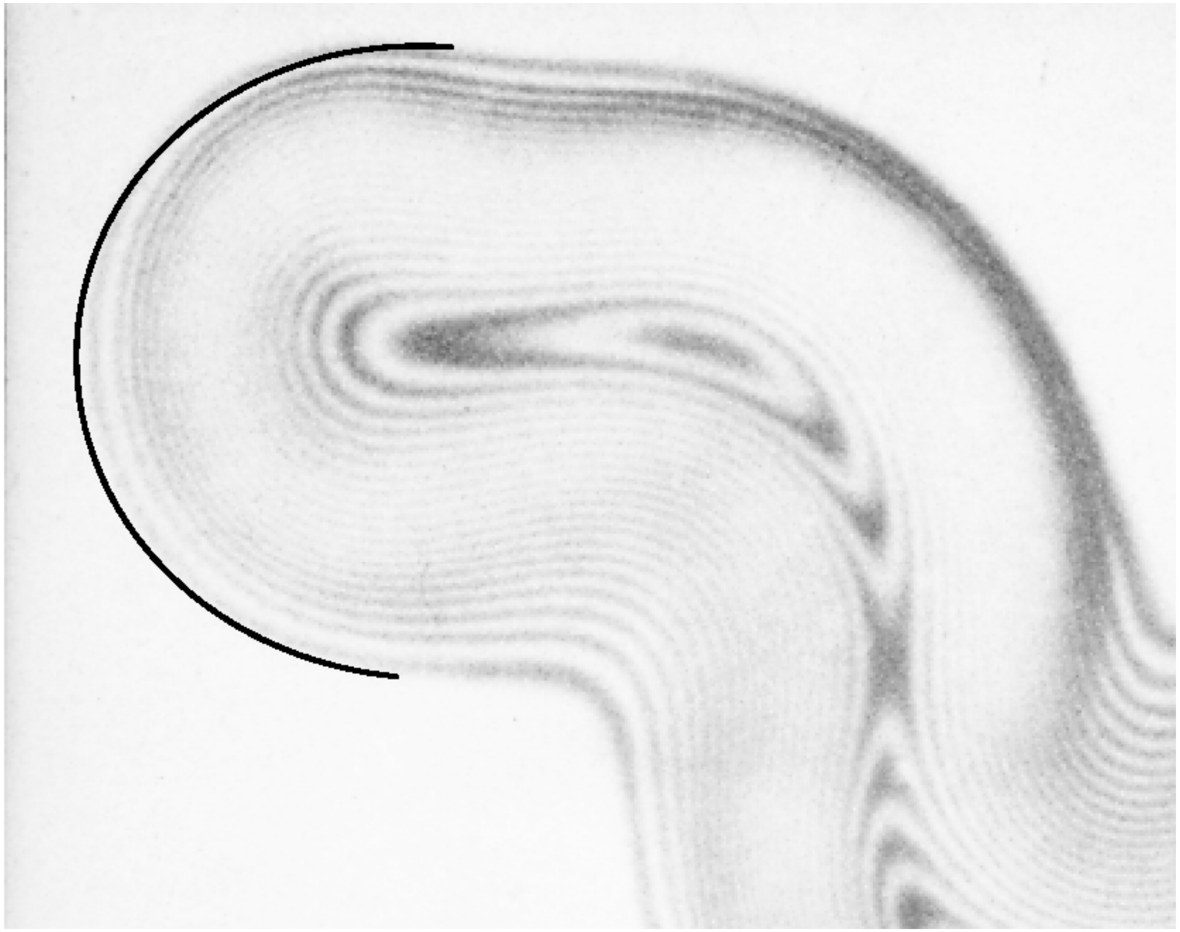
Experimental validation



Comparison of telephone cord in a Si/SiO2~glass system reported by Thouless (1993) and tip shape predicted by the theory (Gioia and Ortiz, 1997)

"You have delighted us long enough!"

(*Pride and Prejudice*, Jane Austen)