



Model-Free Data-Driven Science: Cutting out the Middleman

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BiGmax (virtual) Workshop 2021
on Big-Data-Driven Materials Science
Max-Planck Research Network, April 15, 2021

Data-Driven Science: Talking points

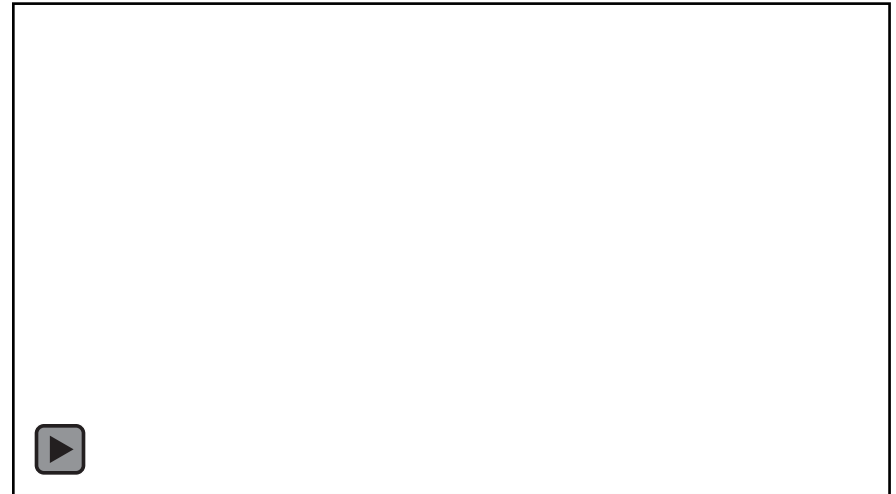
- New emerging paradigm: *Data-Driven Science*
- Why now? *What has changed?*
- How does Data Science intersect with the *physical sciences*? With *experimental science*?
- Is there *intellectual depth* to Data-Driven Science? Are there *opportunities* for fundamental, enduring, contributions?
- Data-Driven Science: *Theory vs. practice*
- Is Data-Driven Science likely to change *engineering practice*? Industry?
- Data-Driven ecosystem, *infrastructure*...

The new data-rich world...

- Material *data is currently plentiful* due to dramatic advances in experimental science (DIC, EBSD, microscopy, tomography...) and multiscale computing (DFT → MD → DDD → SM → Hom)



3D tomographic reconstruction
of particles in battery electrode



3D DIC-measured
internal-strain full-field
compressed PDMS sample

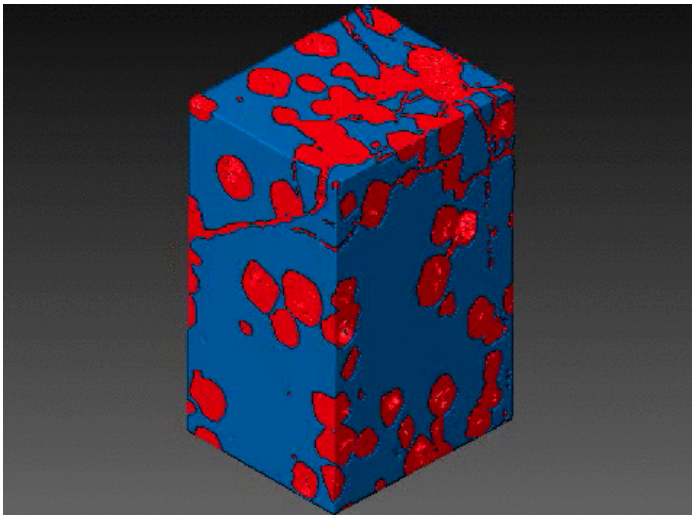
John Lambros, UIUC,

<https://lambros.ae.illinois.edu/moviesimages/>

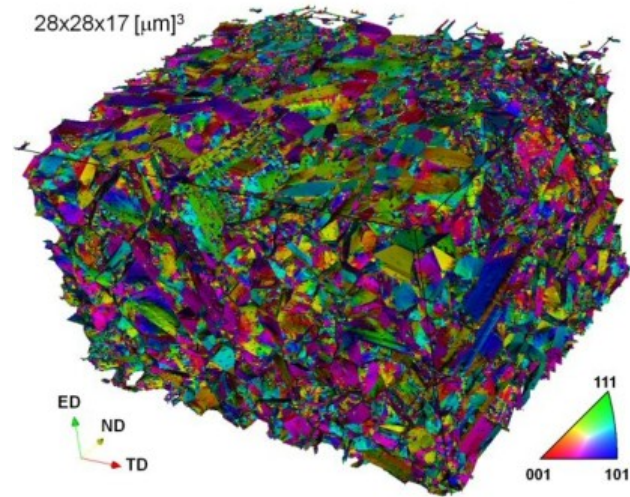
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The new data-rich world...

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Two-phase μ CT analysis of Ti₂AlC/Al composite¹



3D EBSD microstructure in Cu-0.17wt%Zr after ECAP²

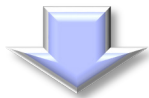
¹Hanaor *etal*, *Mater Sci Eng A*, **672** (2019) 247.

²Khorashadizadeh, *Adv Eng Mater*, **13** (2011) 237.

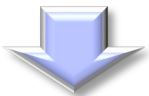
Adapting to a new data-rich world...

Classical Model-Based Computational Science...

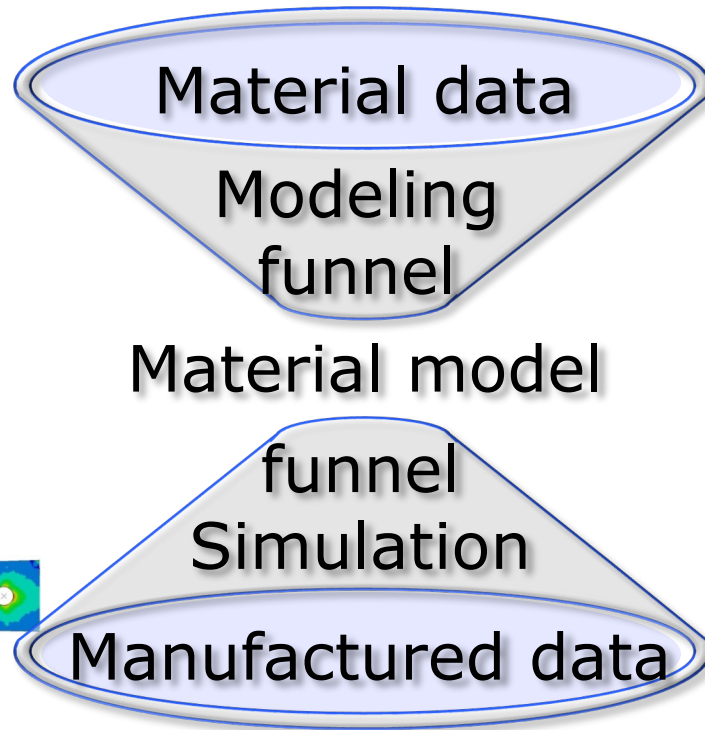
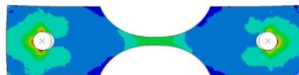
Modern
microscopy
generates
massive
data sets



$$\sigma = \mathbb{C}\epsilon$$



*Modelling
entails
massive
loss of
information!*



Adapting to a new data-rich world...

- Modeling entails *massive loss of information* from material data sets, inserts *uncertainty*
- Modeling is *ad hoc, open ended, ill-posed*
- There is *no theory* that determines models from first principles to a desired level of accuracy
- Modeling requires *heuristics and intuition*: Models are *only as good as the modeler's* physical intuition, empirical knowledge
- *Deep Learning (ANN) – hype = piecewise-linear regression* (cf., e.g., Gilbert Strang, 2019)
- Can a more *direct connection between data and prediction* be forged? (*the data, all the data, nothing but the data!*) How?

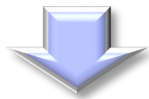
Adapting to a new data-rich world...

Classical Model-Based
Computational Science...

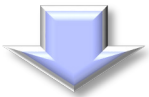


*Alternative:
Model-Free
Data-Driven
Computing!*

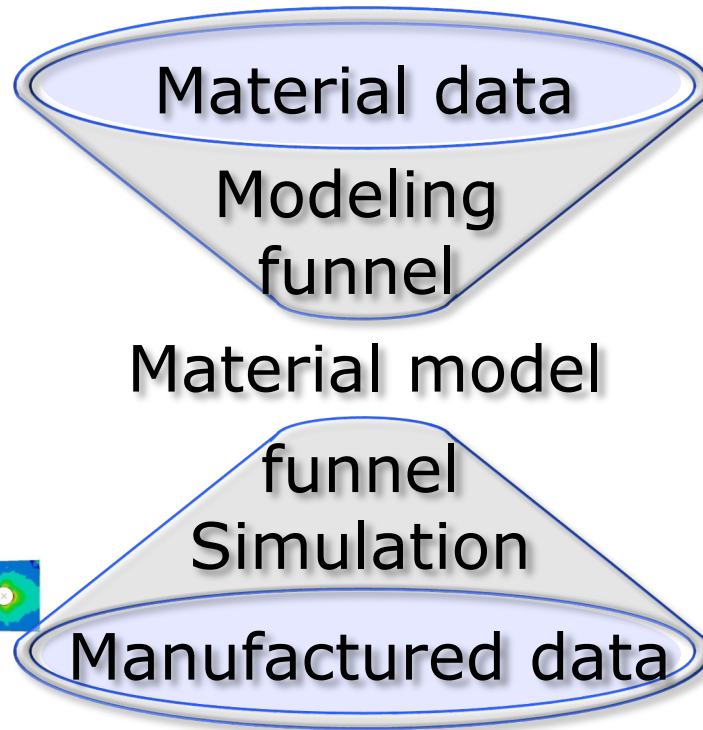
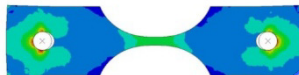
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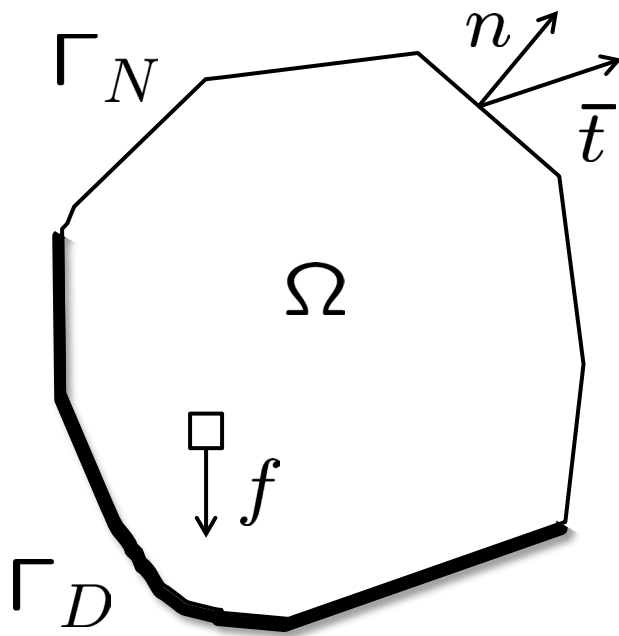


Set/solve
problems
directly
from data!

*Eliminate
modeling
bottleneck!*

Differential structure of field equations

- Anatomy of a *field-theoretical* STEM problem:



- i) Kinematics + Dirichlet BC:

$$\left. \begin{aligned} \epsilon(u) &= 1/2(\nabla u + \nabla u^T) \\ u &= \bar{u}, \quad \text{on } \Gamma_D \end{aligned} \right\}$$

- ii) Equilibrium + Neumann BC:

$$\left. \begin{aligned} \operatorname{div} \sigma + f &= 0 \\ \sigma n &= \bar{t}, \quad \text{on } \Gamma_N \end{aligned} \right\}$$

iii) *Material law*: $\sigma(x) = \sigma(\epsilon(x))$

Differential structure of field equations

- Anatomy of a *field-theoretical* STEM problem:

Universal laws!
 Material
 independent!
 Exactly known!
 Uncertainty-free!
 Similarly:
 Maxwell, Einstein,
 Schrödinger
 Mass transport...

i) Kinematics + Dirichlet BC:

$$\left. \begin{aligned} \epsilon(u) &= 1/2(\nabla u + \nabla u^T) \\ u &= \bar{u}, \quad \text{on } \Gamma_D \end{aligned} \right\} \delta!$$

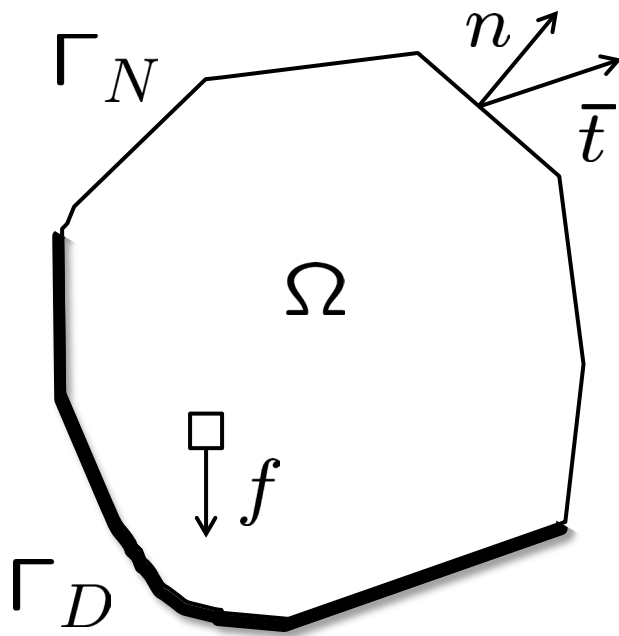
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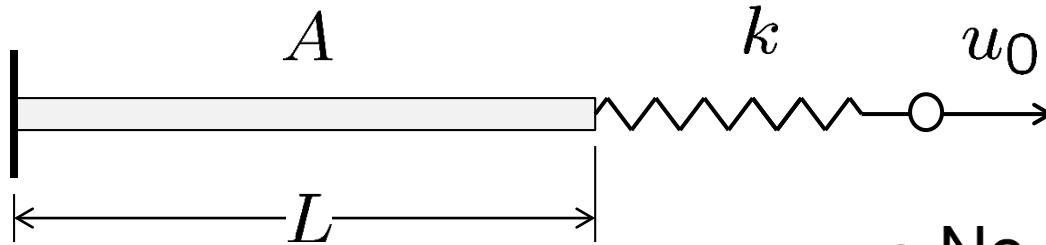
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Unknown! Epistemic uncertainty!



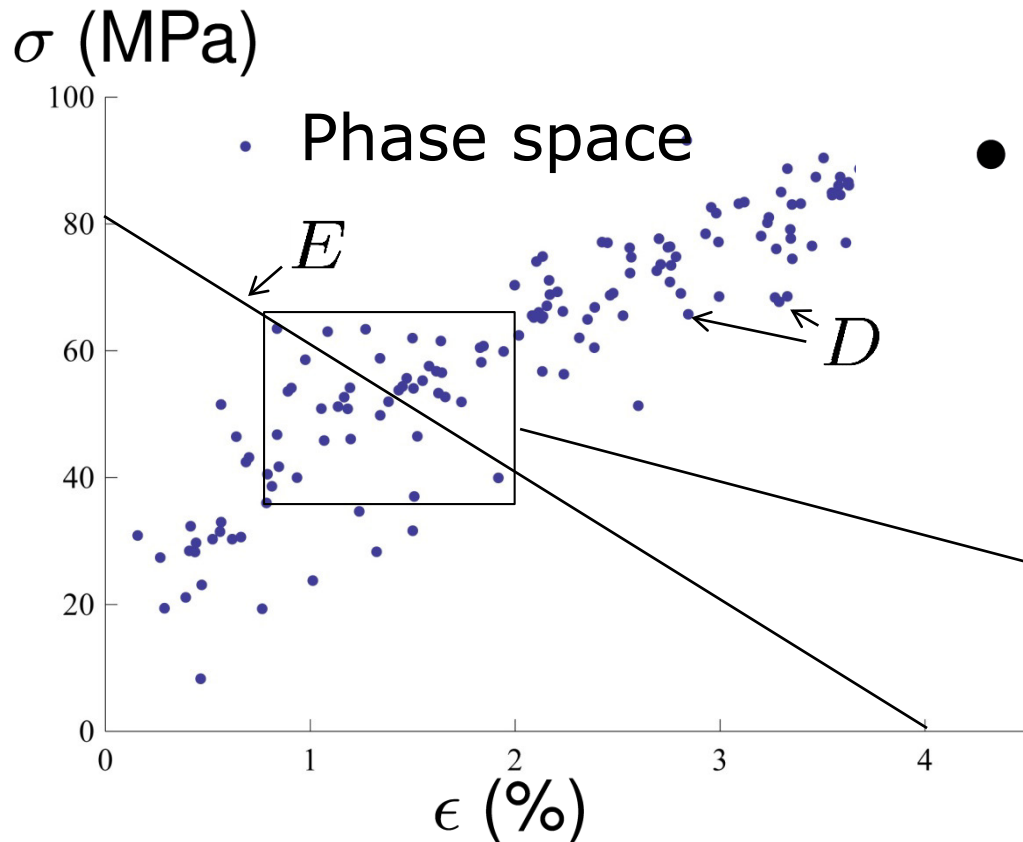
Elementary example: Bar and spring



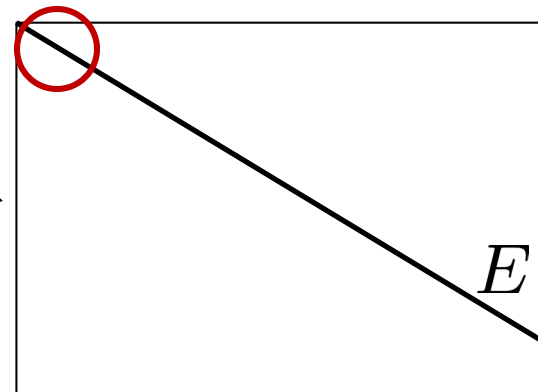
- No classical solutions!

$$D \cap E = \emptyset$$

- Data-driven solution:



$$\min_{z \in E} \text{dist}(z, D)$$



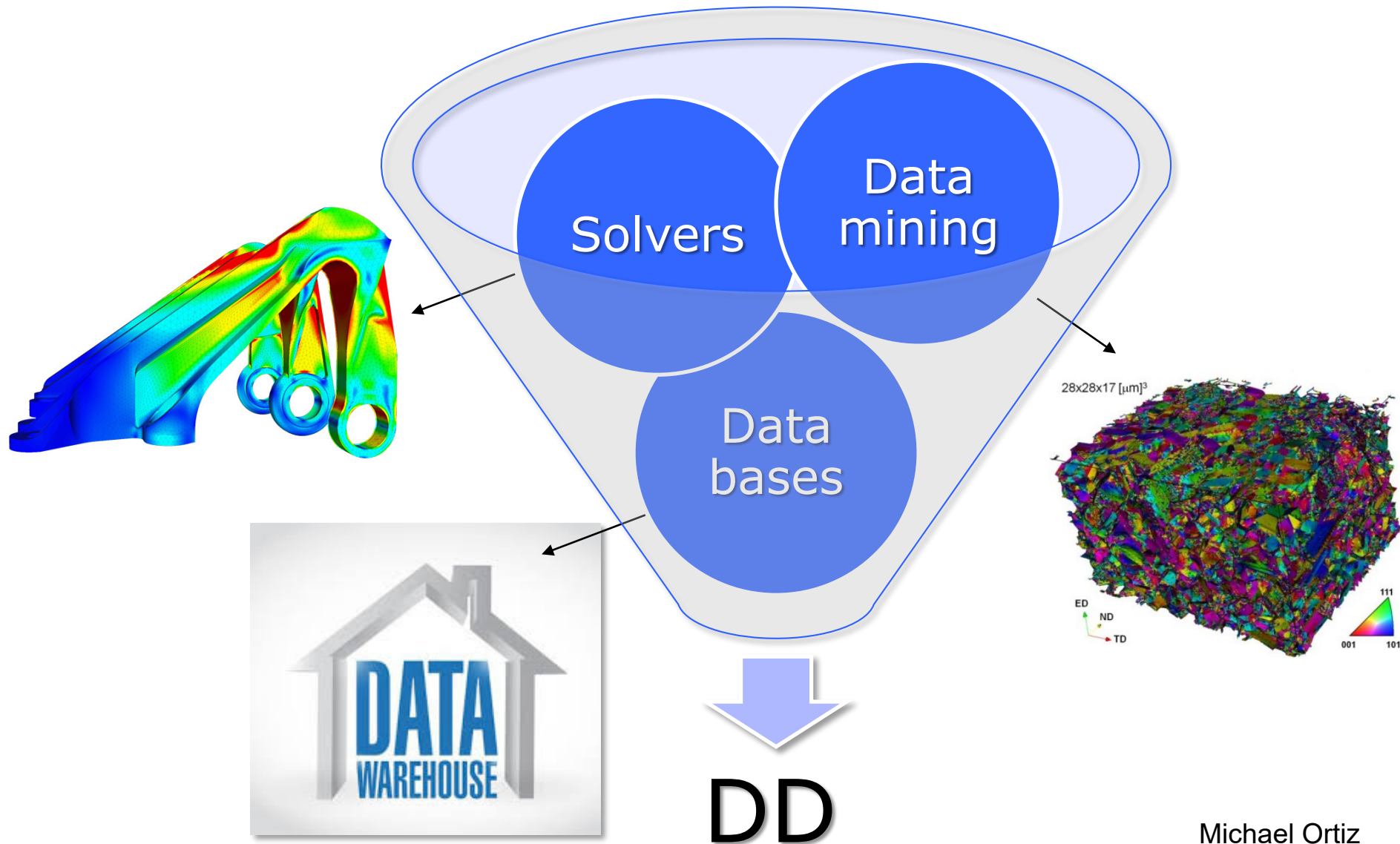
The general Data-Driven (DD) problem

- The *Data-Driven paradigm*¹: Given,
 - $D = \{\text{fundamental material data}\},$
 - $E = \{\text{compatibility} + \text{equilibrium}\},$Find: $\operatorname{argmin}\{d(z, D), z \in E\}$
- *The aim of Data-Driven analysis is to find the admissible state (compatibility and equilibrium) closest to the material data set*
- Raw *fundamental material data* (stress and strain) is used (unprocessed) in calculations
- *No material modeling, no loss of information (the data, all the data, nothing but the data!)*

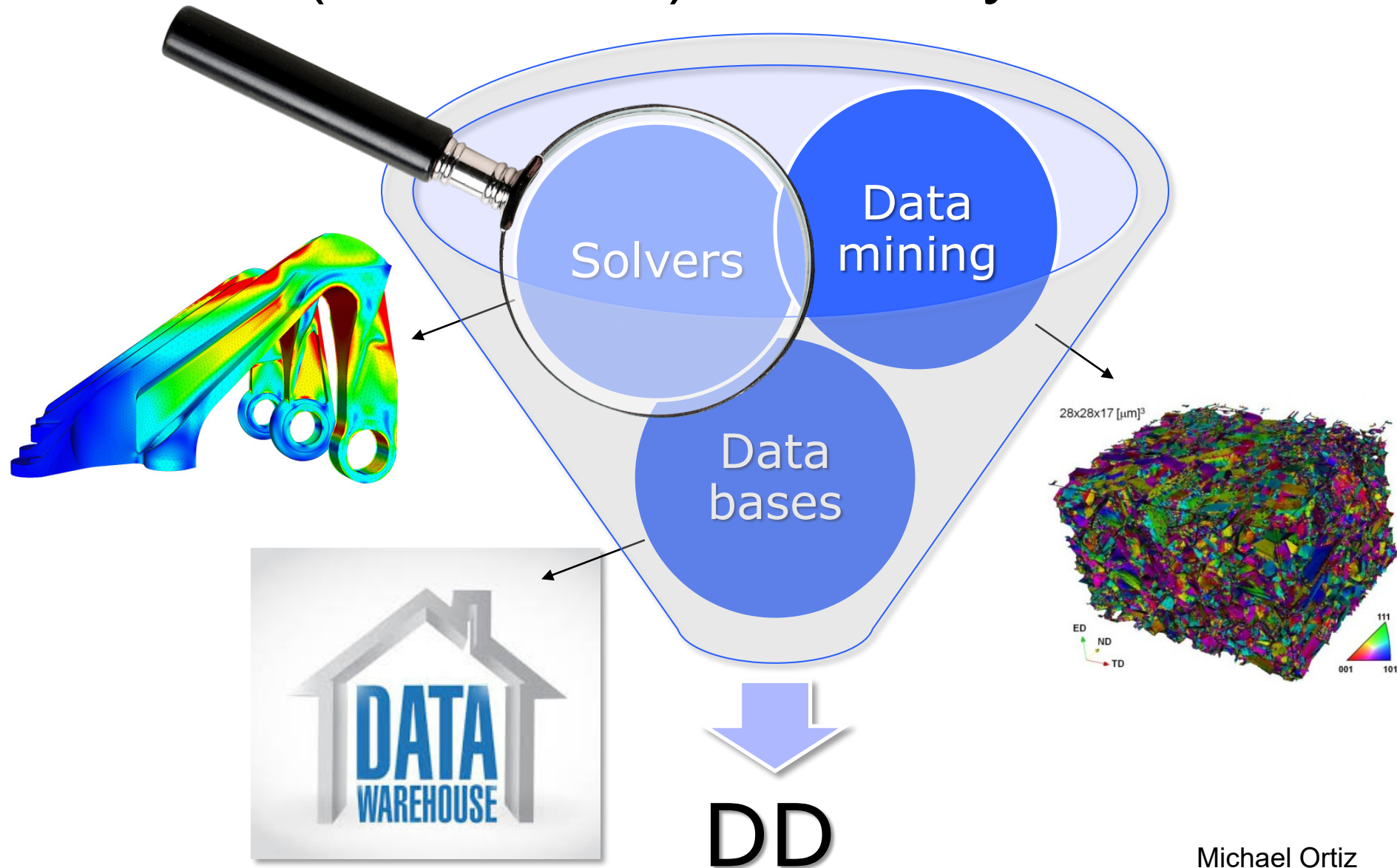
¹T. Kirchdoerfer and M. Ortiz (2015) arXiv:1510.04232.

¹T. Kirchdoerfer and M. Ortiz, *CMAME*, **304** (2016) 81–101

The (model-free) DD ecosystem...

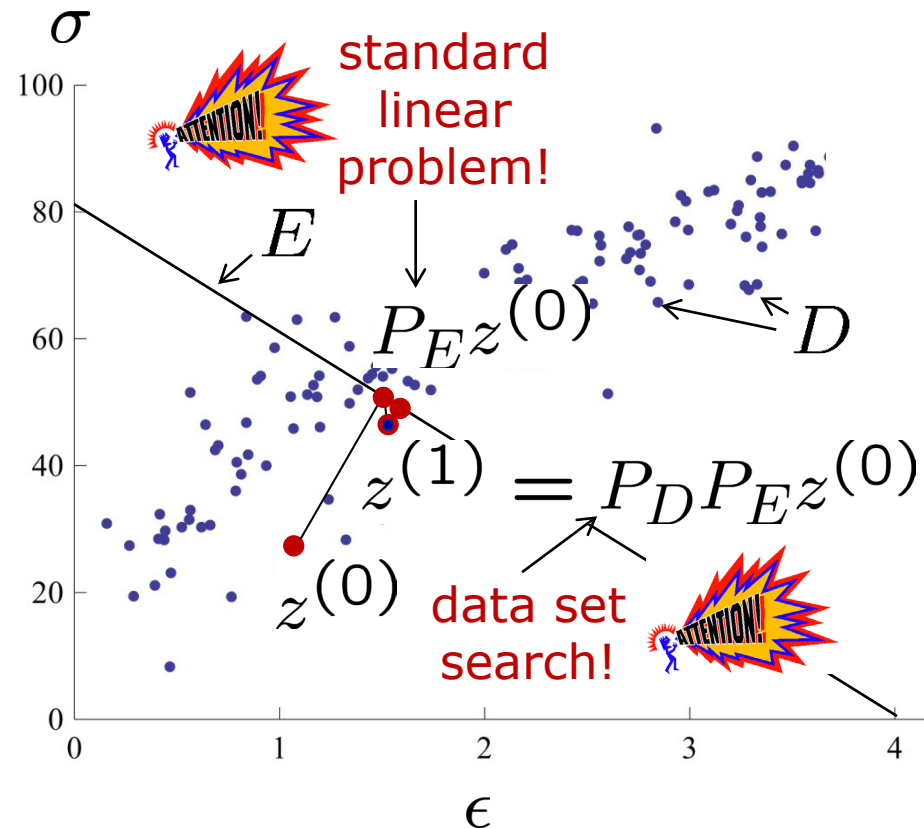
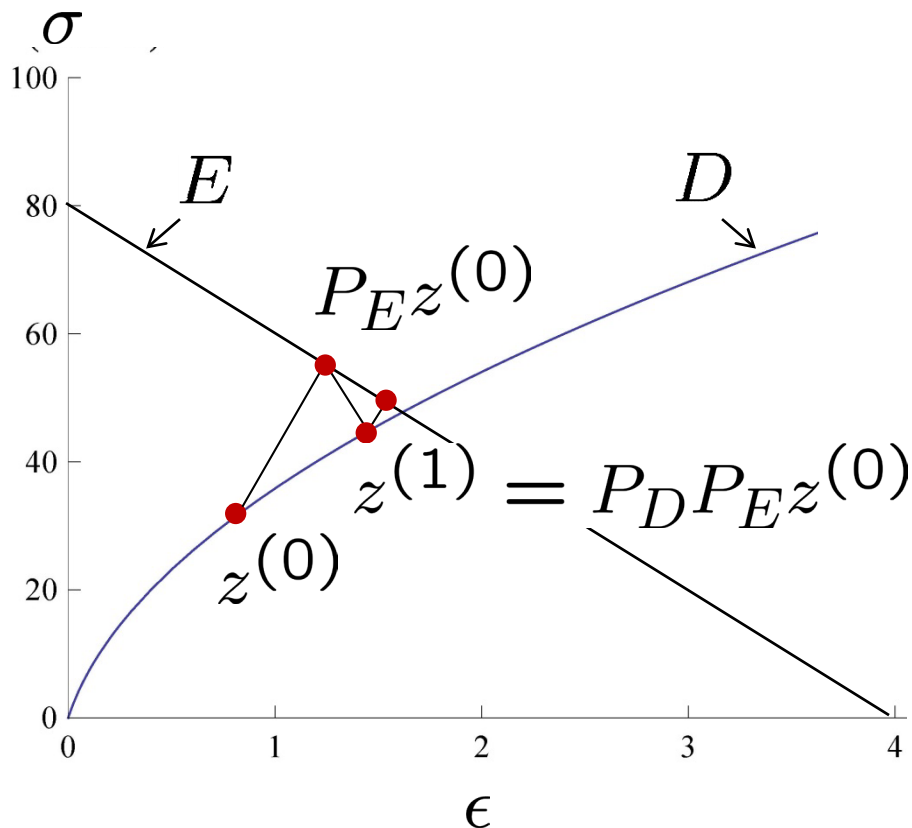


The (model-free) DD ecosystem...



DD solvers: Fixed-point iteration

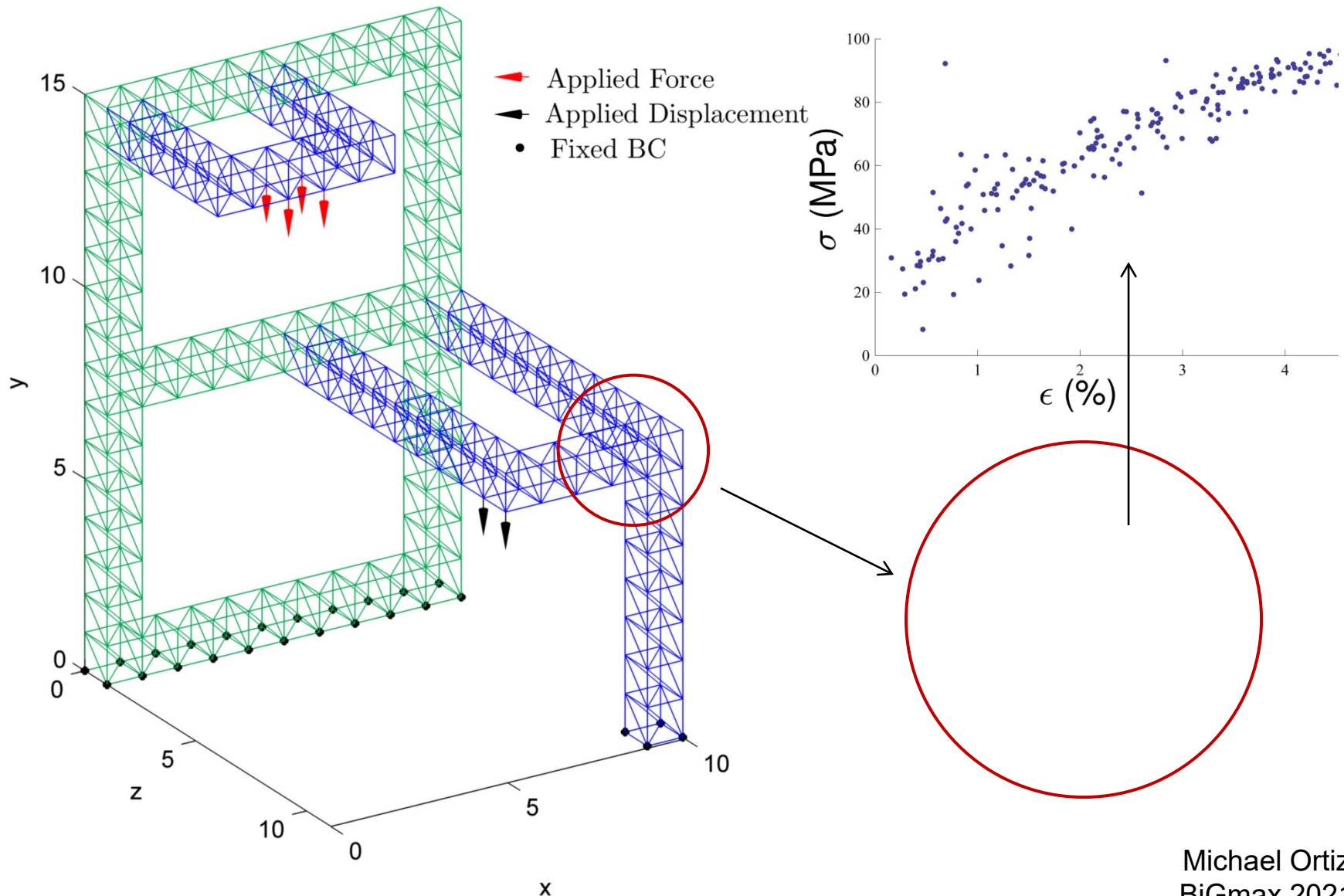
- Find: $\operatorname{argmin}\{d(z, D), z \in E\}$
- Fixed-point iteration¹: $z^{(k+1)} = P_D P_E z^{(k)}$



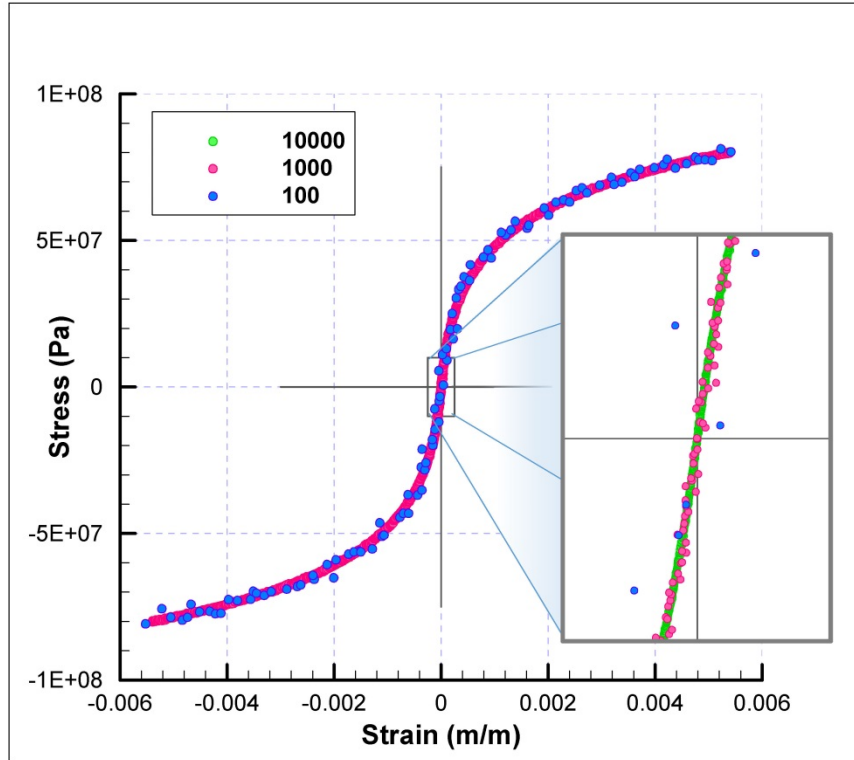
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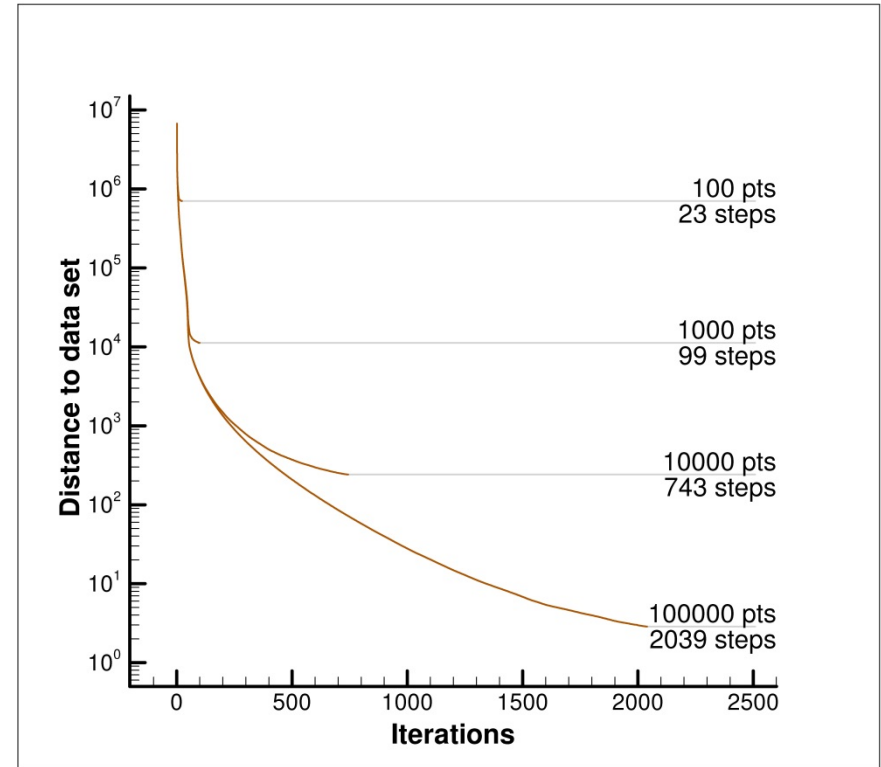
Example: 3D Truss



Convergence of fixed-point solver

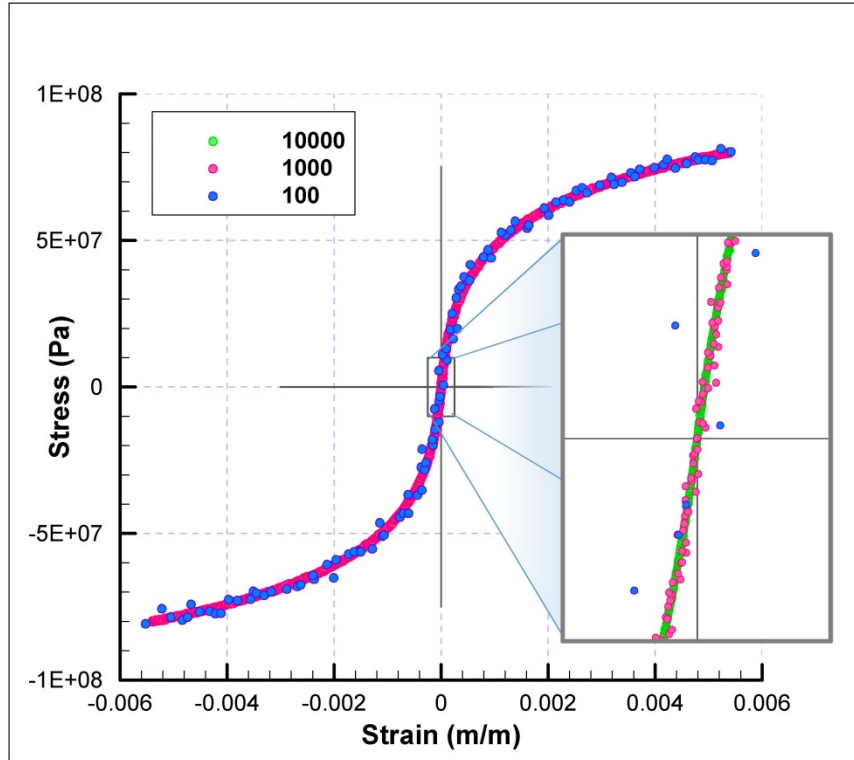


Randomized material-data
sets of increasing size

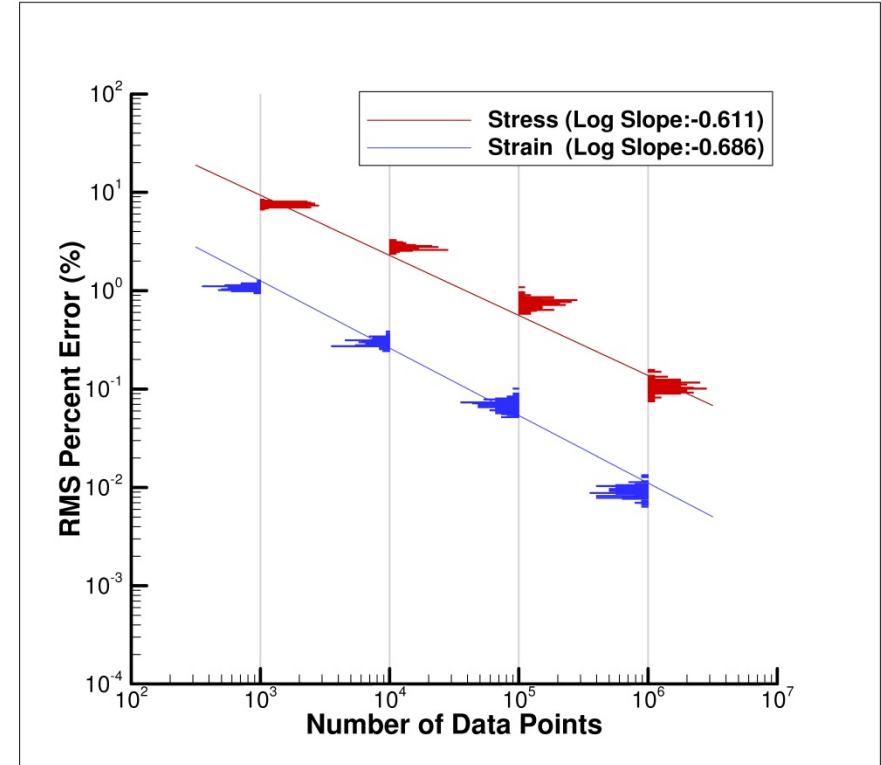


Convergence of
fixed-point iteration

Convergence of fixed-point solver



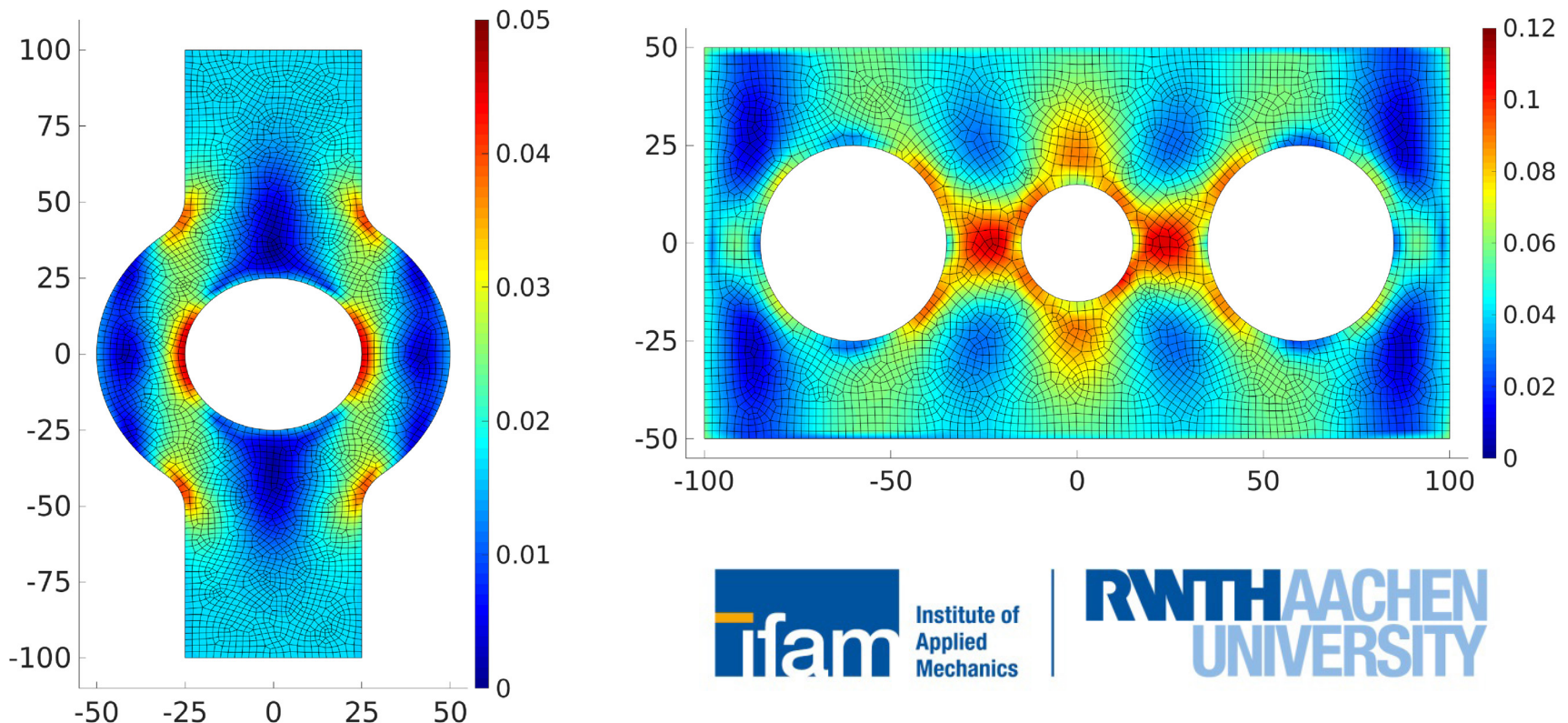
Randomized material-data
sets of increasing size
and decreasing scatter



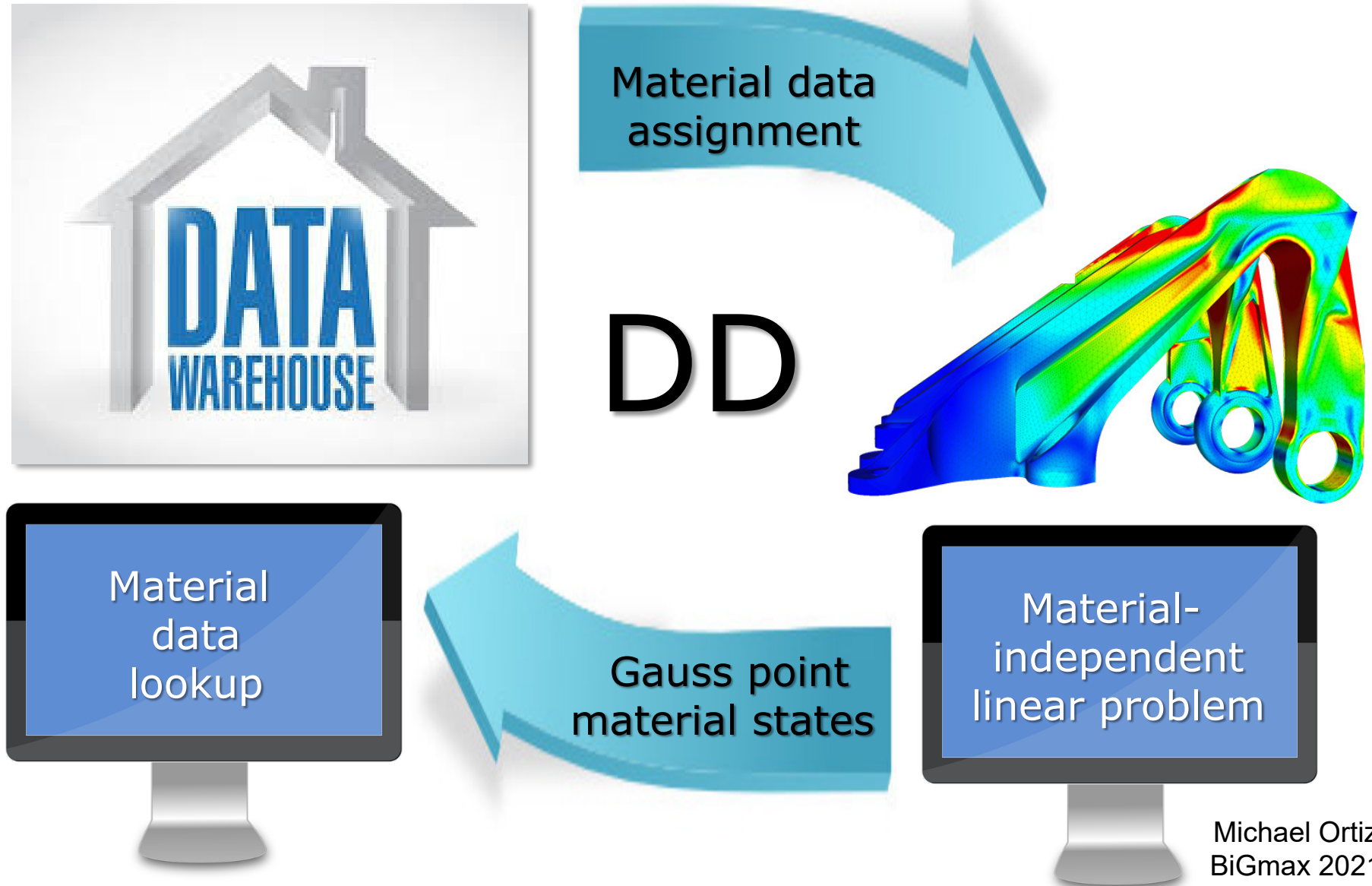
Convergence with
respect to data set

Data-Driven solvers and FE analysis

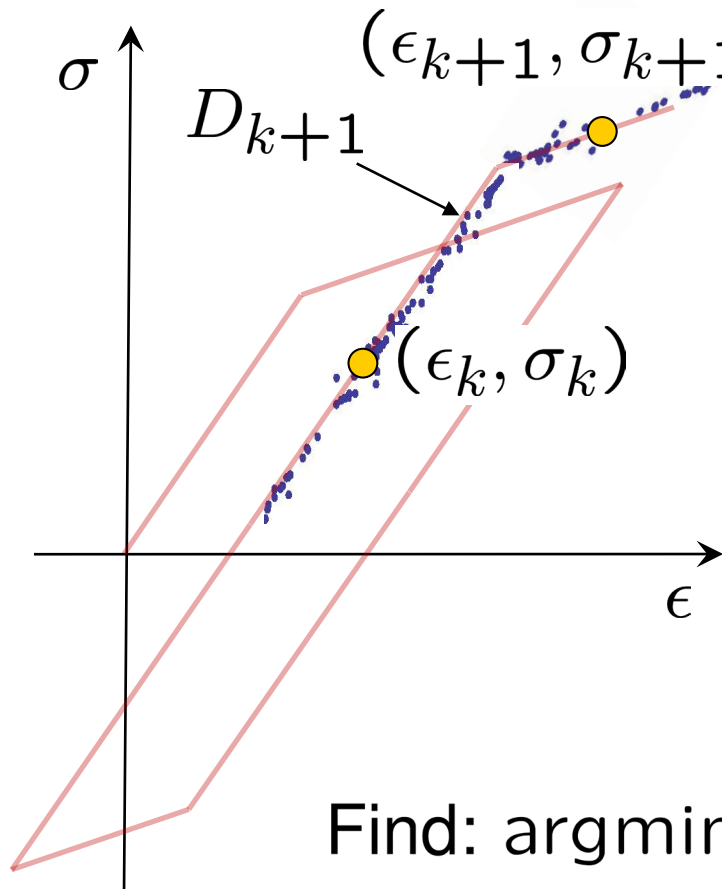
- (Model-free) Data-Driven solvers can be easily implemented within a finite-element framework



The DD information flow



Data-Driven inelasticity



• Material set representation:
 $D_{k+1} = \{(\epsilon_{k+1}, \sigma_{k+1}) : \text{history}\}$

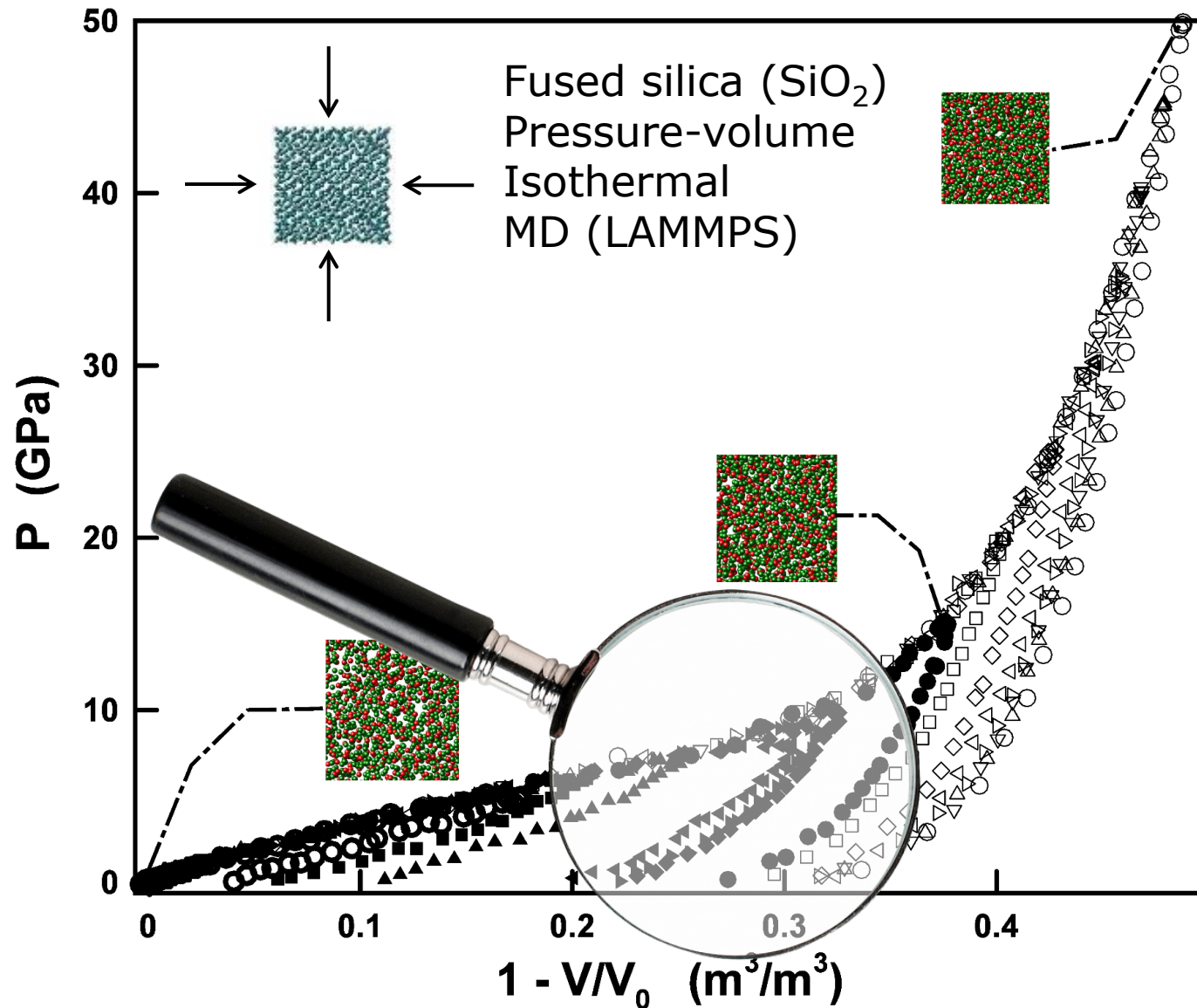
- Need *material history data*! (from material testing along selected loading paths...)
- History data must provide adequate *path coverage*...
- Data-driven problem:

Find: $\operatorname{argmin}\{d(z, D_{k+1}), z \in E_{k+1}\}$

time dependent!

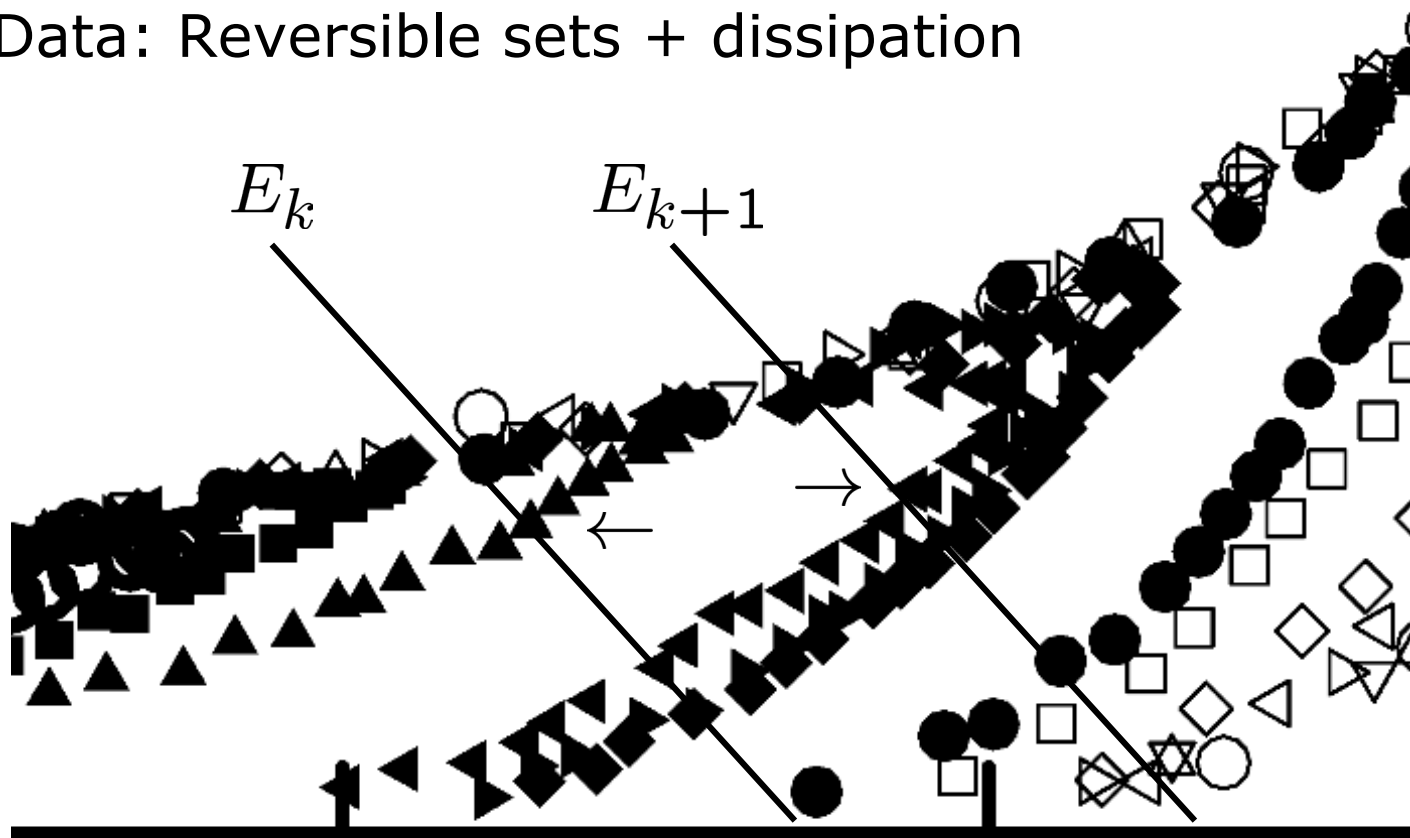


Data-Driven inelasticity



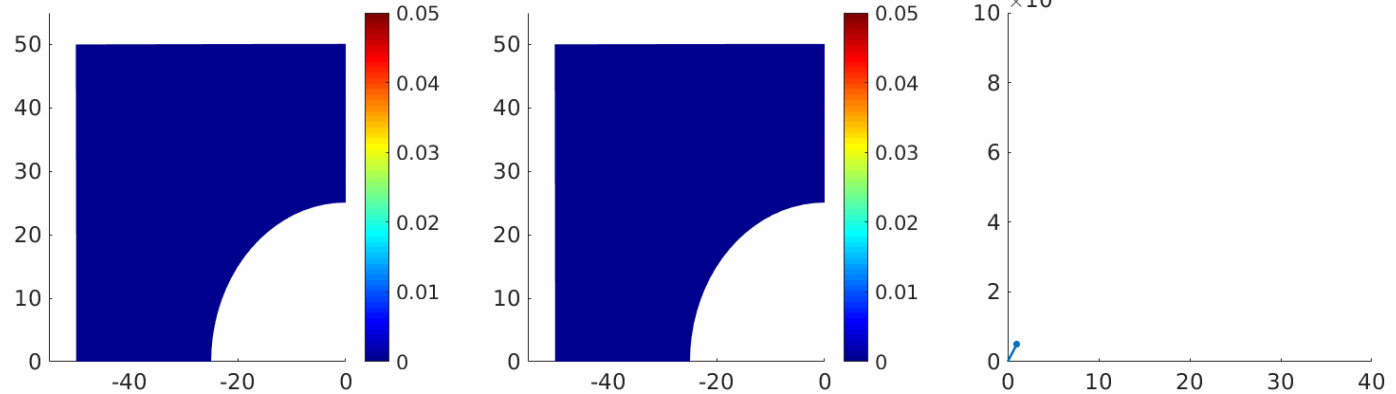
Data-Driven inelasticity

- Phase space: Add temperature and entropy
- Field eqs: Add energy conservation + second law
- Data: Reversible sets + dissipation

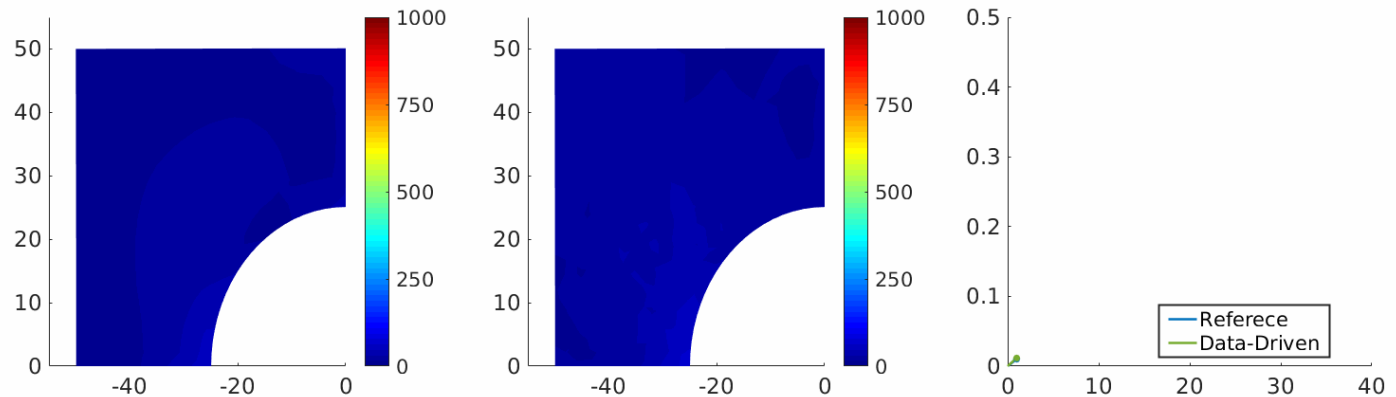


Data-Driven plasticity

Strain (von Mises)

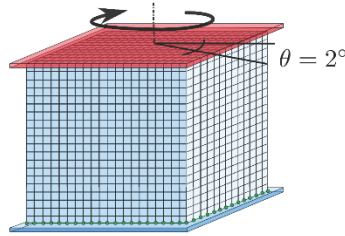
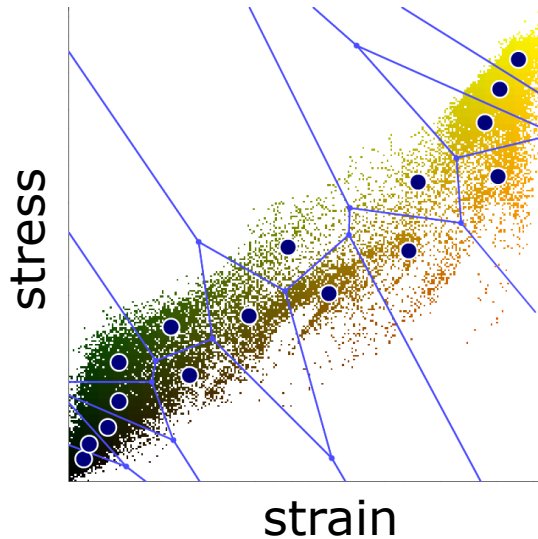


Stress (von Mises)

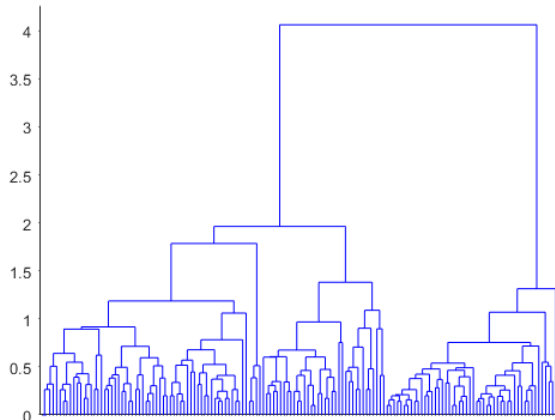
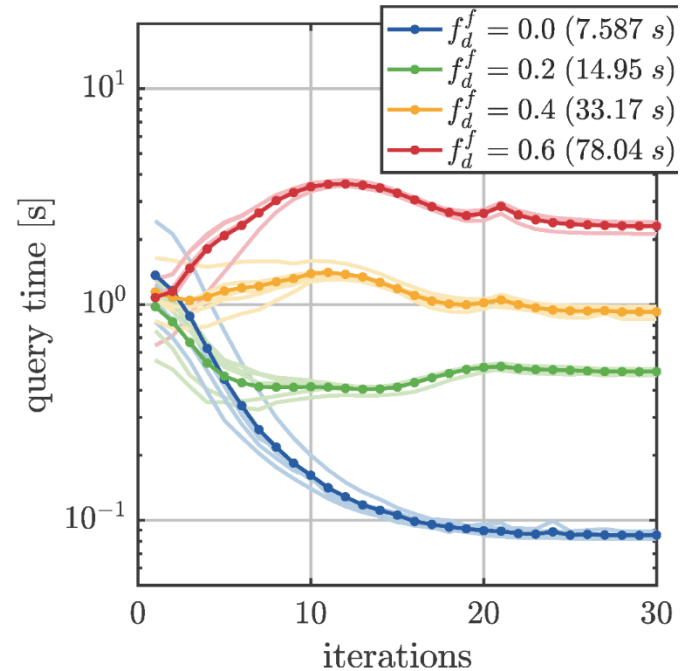


(R. Eggersmann & S. Reese, 2019)

Connection to Machine Learning



Test problem:
Torsion of
20x20x20 cube
64000 mat pts



K-means hierarchical structure

- Material data set: **1 billion points**
- Approx k-means search, 0.1 secs
- **Set-oriented machine learning!**
- We learn the **structure of the data set**
- No regression, **no loss of information!**
- **The data, all the data, nothing but the data!**

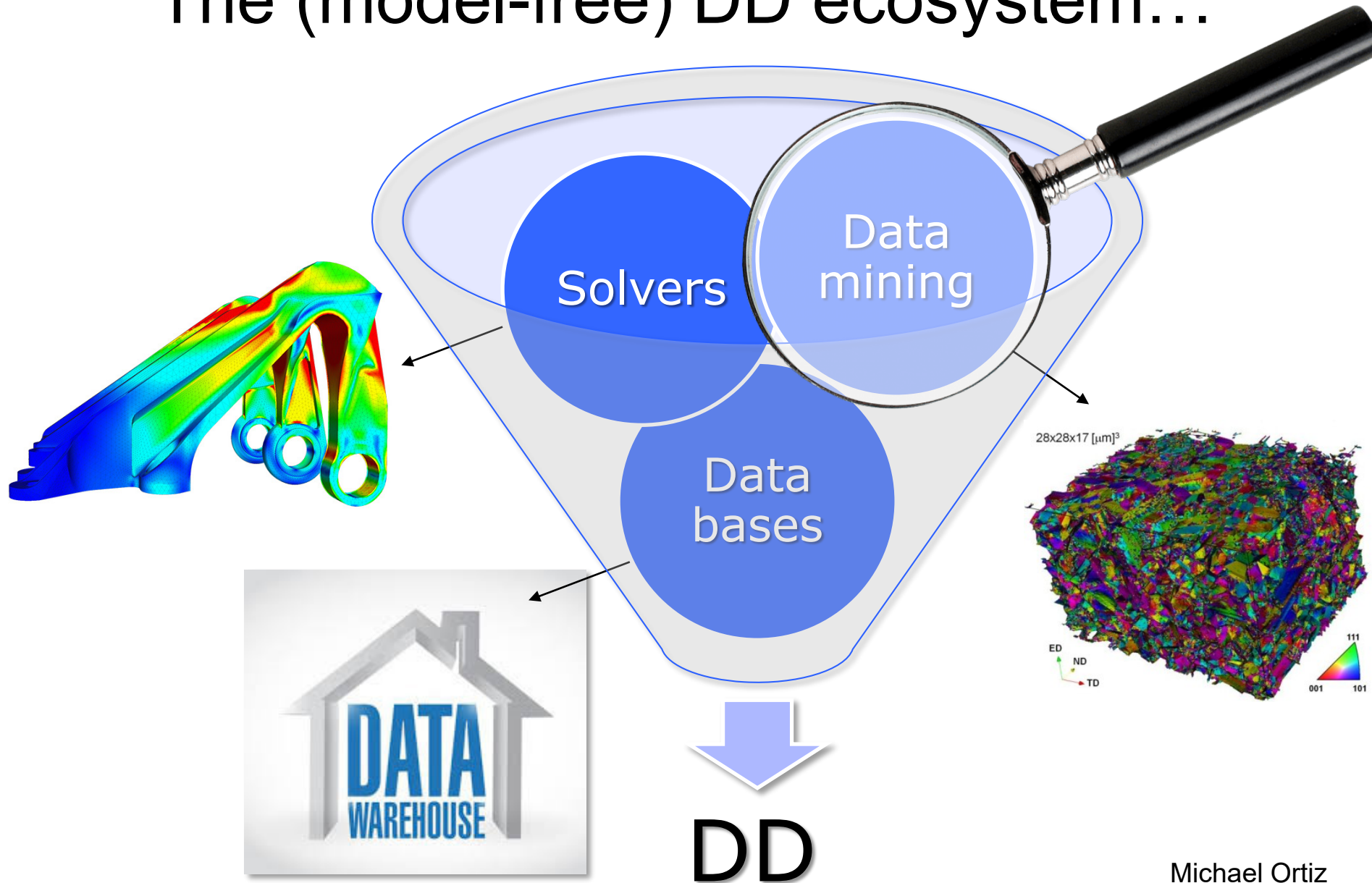
Model-Free Data-Driven (DD) solvers

- Model-Free Data-Driven solvers forge a *direct connection* between material data and prediction
- *No material modeling, no loss of information (the data, all the data, nothing but the data)*
- *Standardization of solvers*: Material-independent linear back-substitutions (geometry+loading FE)
- *Standardization of data*: Fundamental, model-independent data (e.g., stress and strain)
- *Lossless, set-oriented Machine Learning*: graph structure of data sets (e.g., hierarchical clustering, NN trees/graphs...)

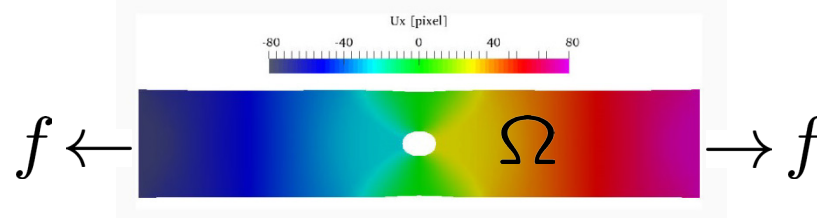
¹T. Kirchdoerfer and M. Ortiz (2015) arXiv:1510.04232.

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The (model-free) DD ecosystem...



DD material identification (DDMI)



- Full-field measurements (DIC): $(\epsilon'(x), f(x))$
- *Stress field cannot be measured directly!*
- *Density of states* in phase space:

$$\rho(\epsilon, \sigma) = \frac{1}{|\Omega|} \int_{\Omega} \delta(\epsilon - \epsilon'(x), \sigma - \sigma'(x)) dx$$

- Maximize *information-theoretical entropy*: $S(\rho)$
subject to *equilibrium*: $\text{div } \sigma'(x) + f(x) = 0$
- Solve (inverse) problem using *DD k-means*

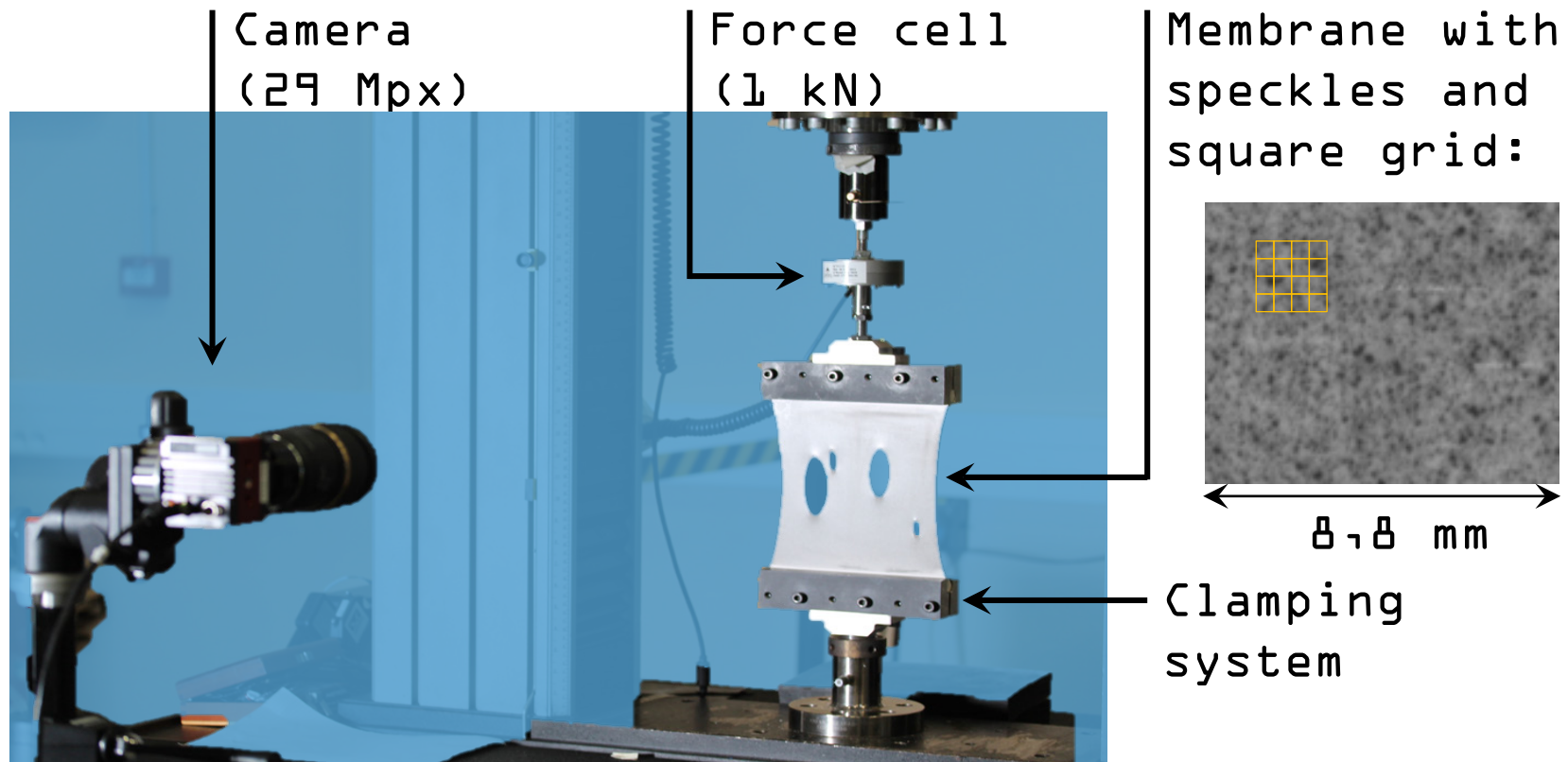
J. Rethore, HAL Id: hal-01454432, Feb. 2017.

J. Rethore and A. Leygue, HAL Id: hal-01452494, Feb. 2017.

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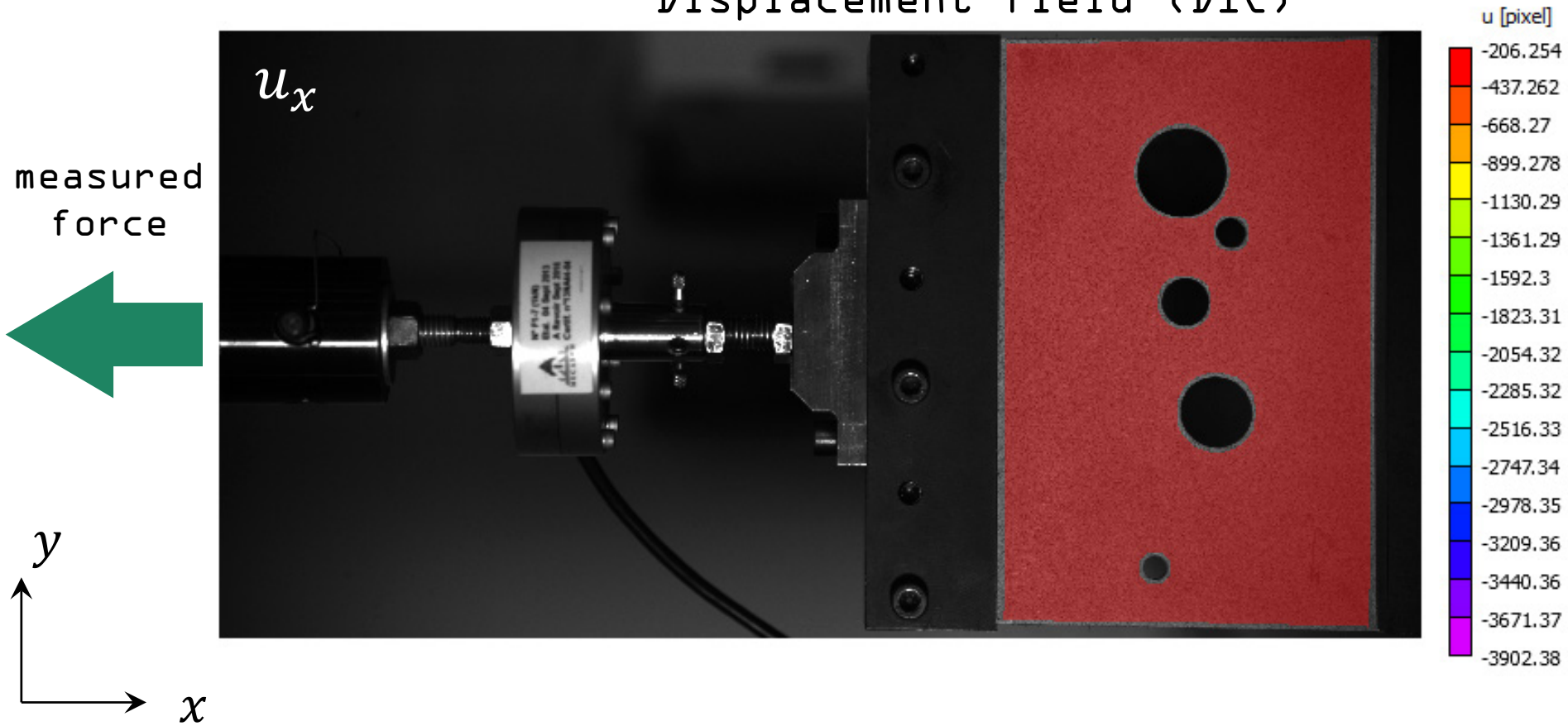
DDMI at Central Nantes

- Tests on perforated silicone sheets

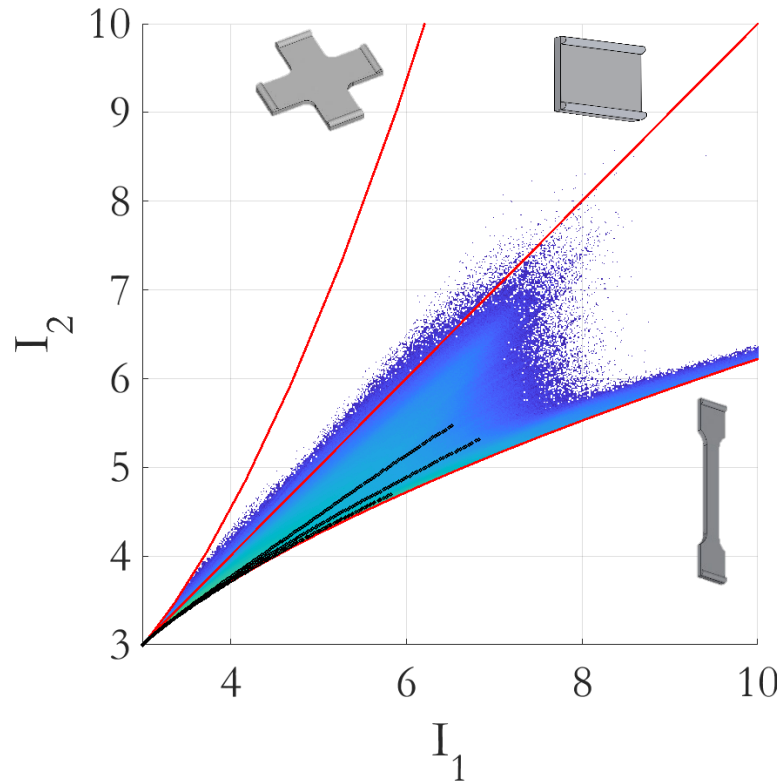


DDMI at Central Nantes

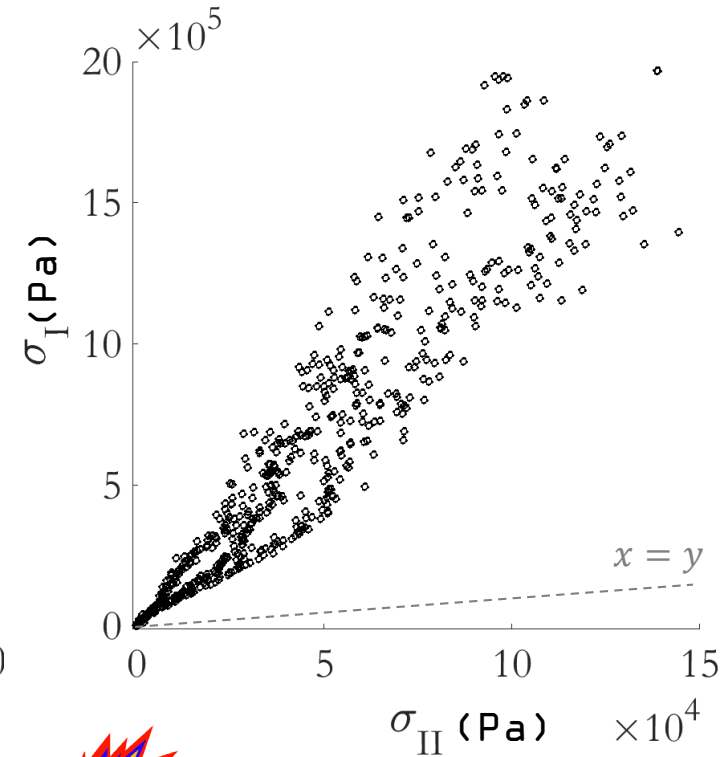
Displacement field (DIC)



DDMI at Central Nantes



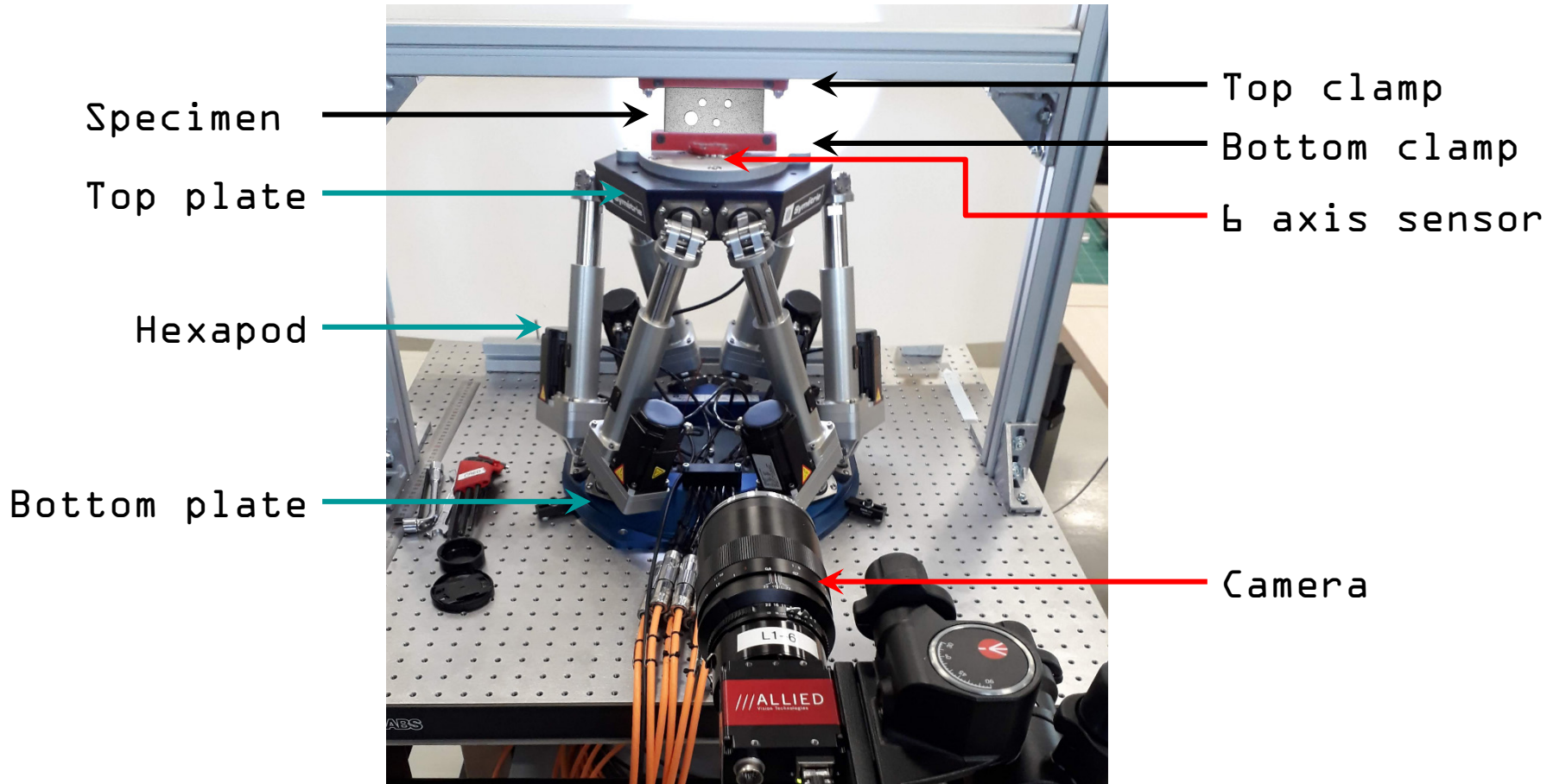
Strain data



Stress data

DDMI at Central Nantes

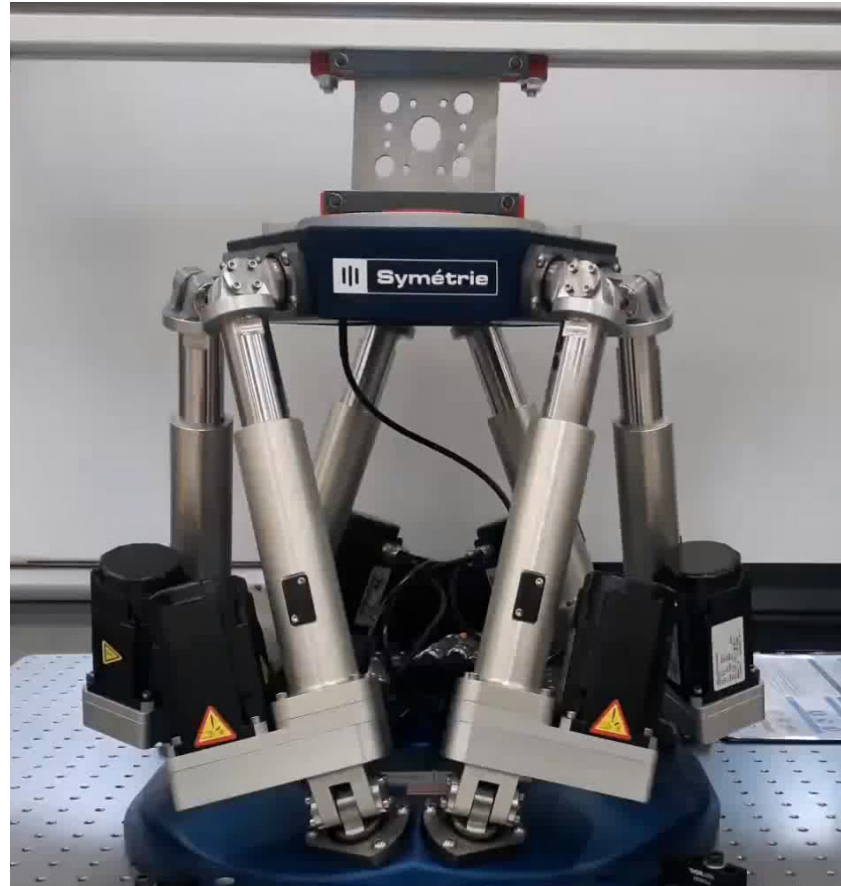
- Hexapod testing machine



M. Dalémat, M. Coret, A. Leygue and E. Verron,
Mechanics of Materials, **136** (2019) 103087.

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DDMI at Central Nantes



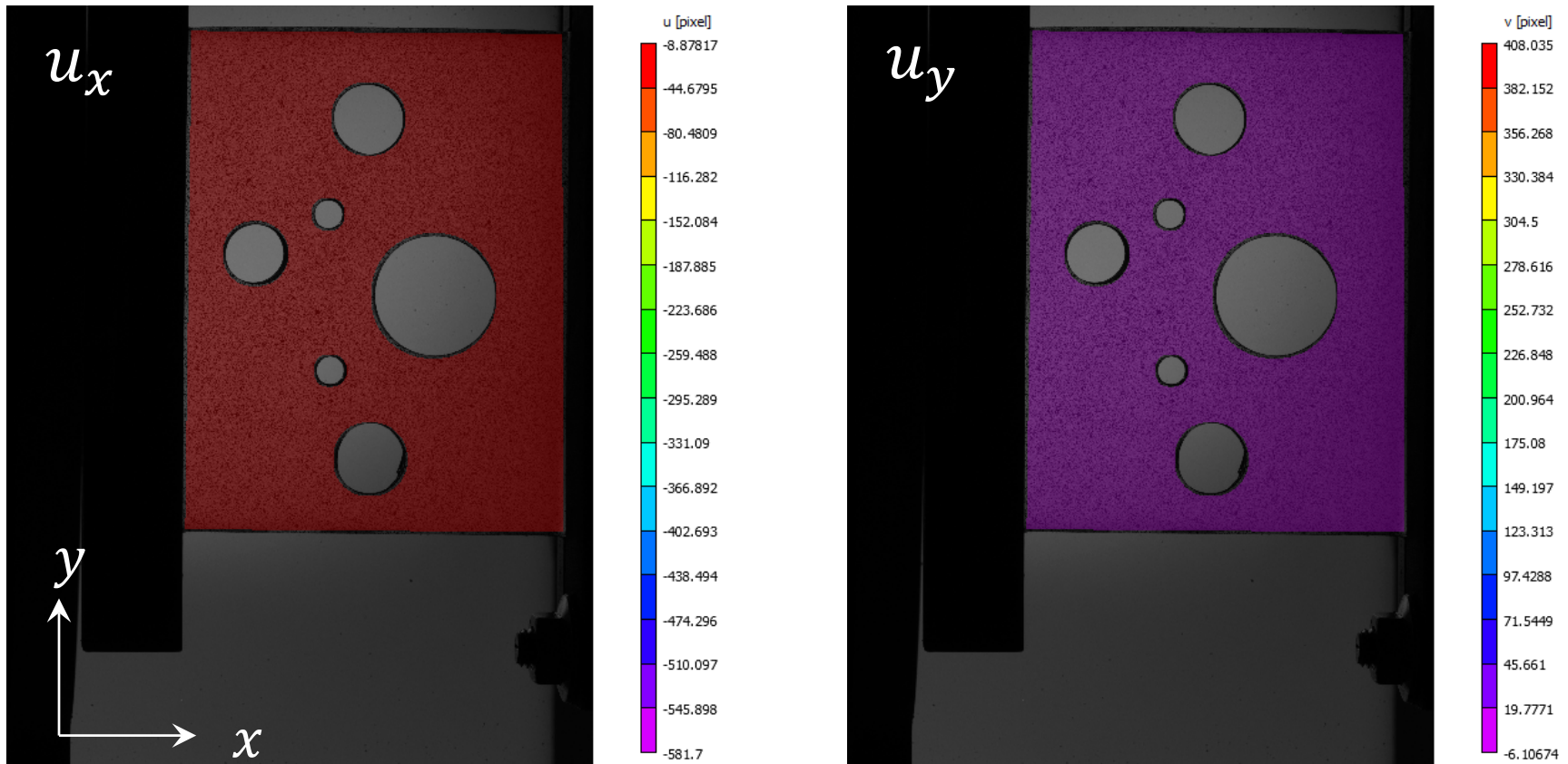
Hexapod in operation

M. Dalémat, M. Coret, A. Leygue and E. Verron,
Mechanics of Materials, **136** (2019) 103087.

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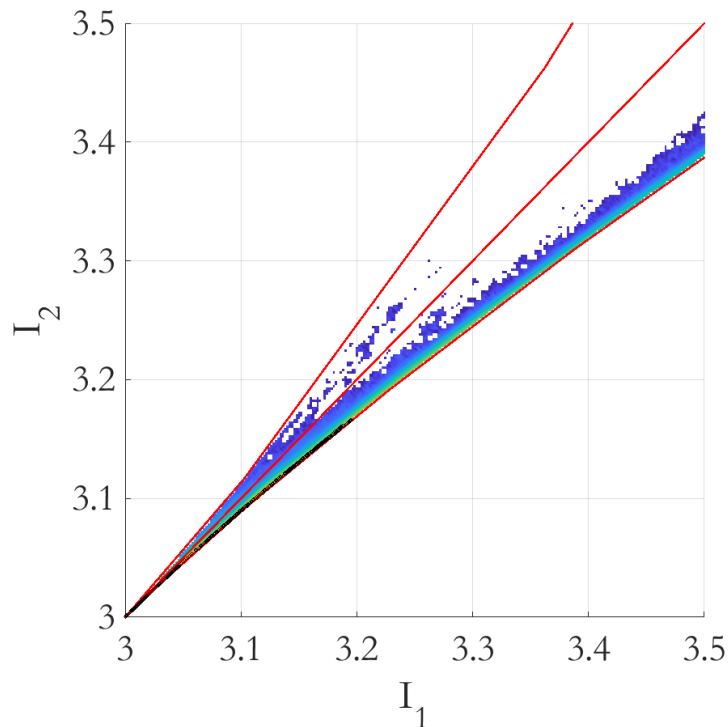
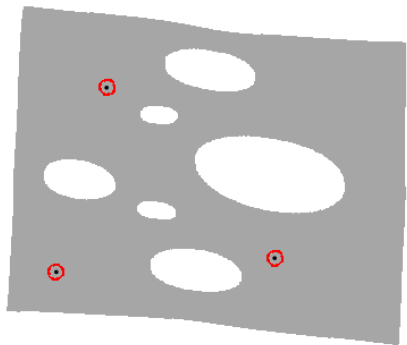
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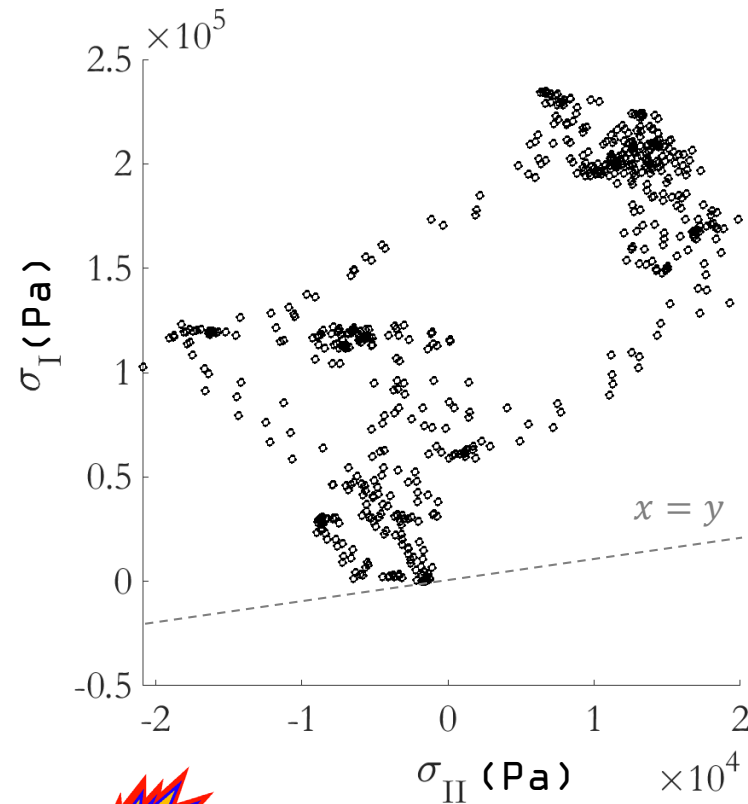
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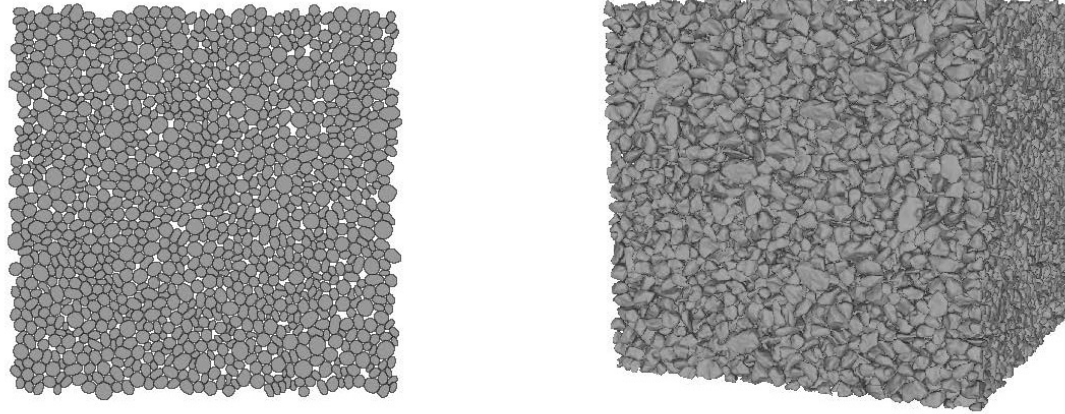


Strain data

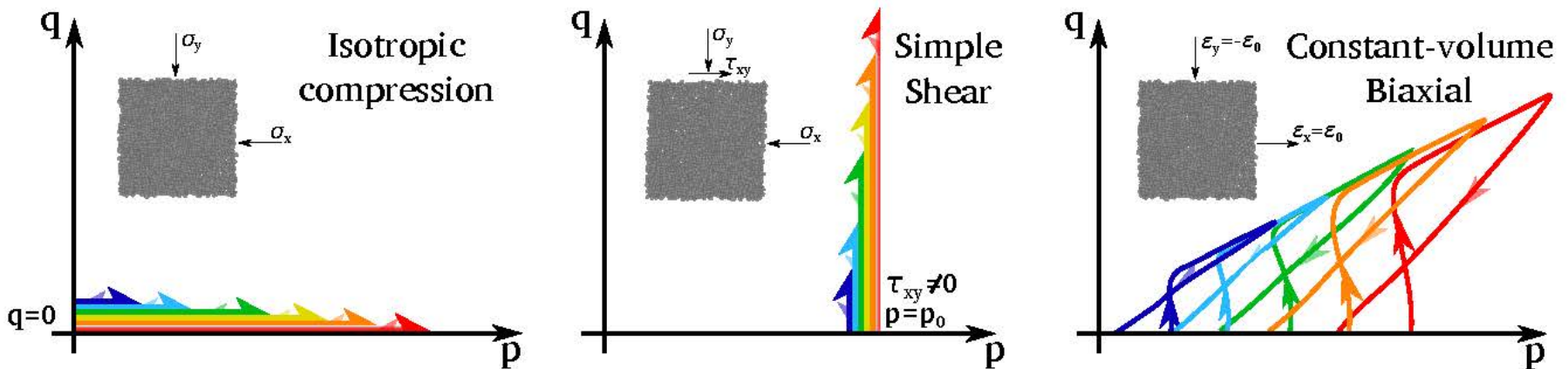


Stress data

Multiscale data mining



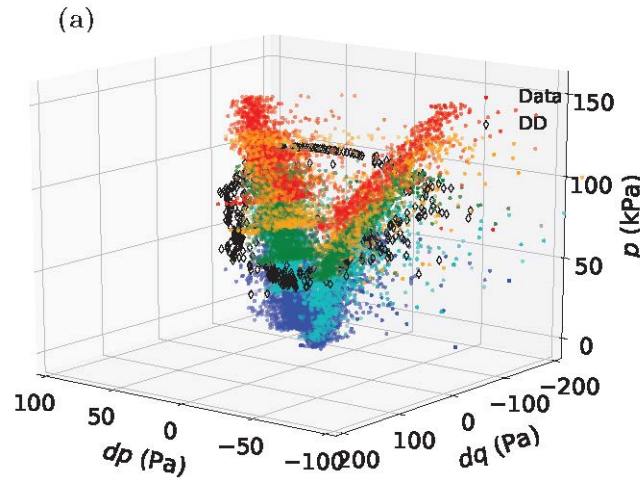
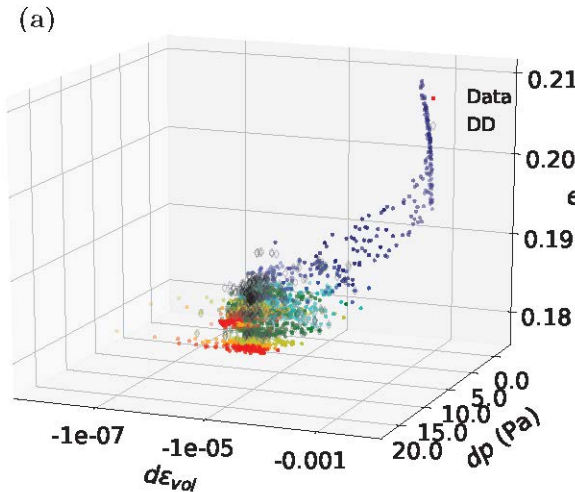
2D and 3D RVE and granular assemblies for LS-DEM analysis



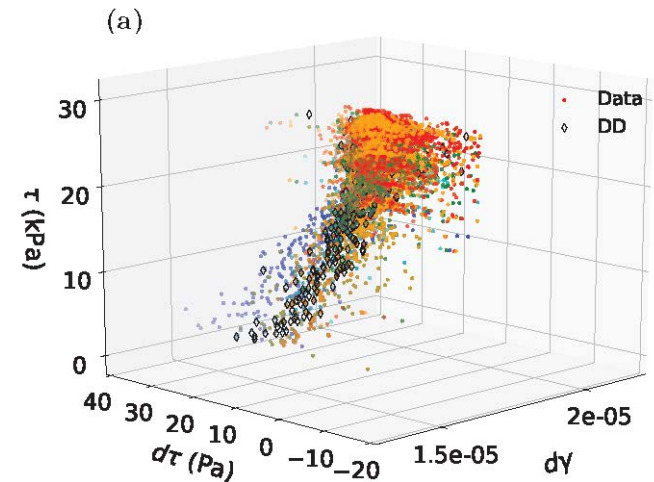
Stress paths for material point simulations and data mining

Multiscale data mining

Isotropic
compression



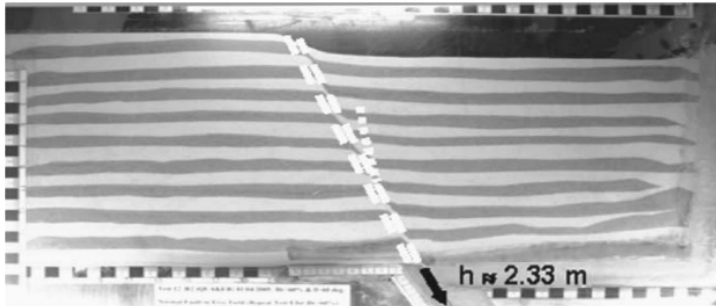
Simple
shear



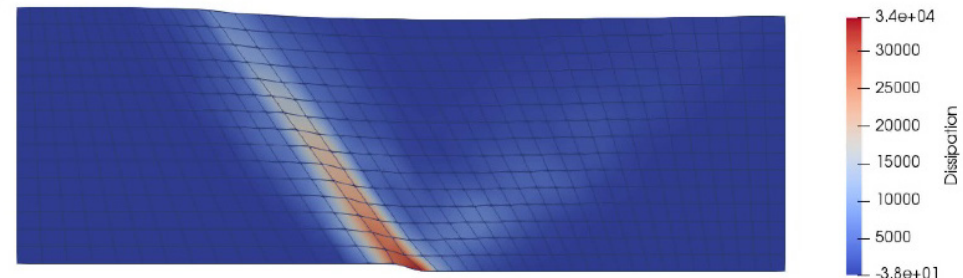
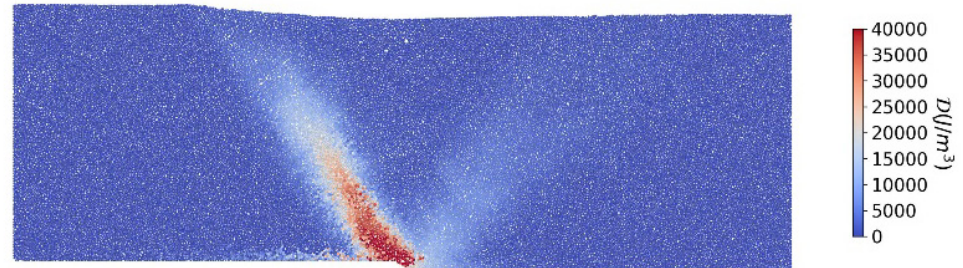
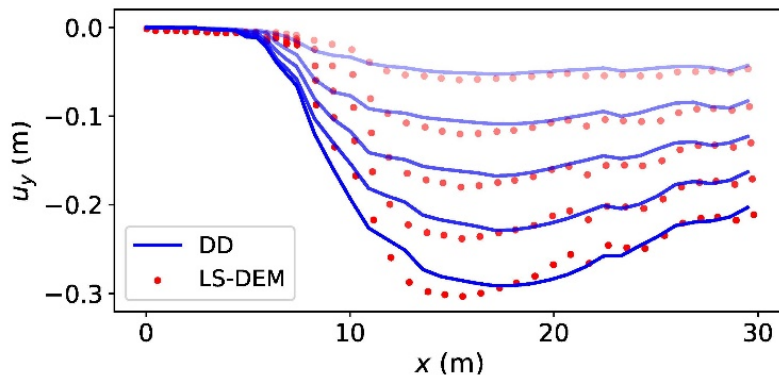
Biaxial
compression

Data computed from
RVE LS-DEM calculations

(Model-free) Data-Driven – Sand



Experimental fault rupture experiment
(Anastapoulos et al., 2007)

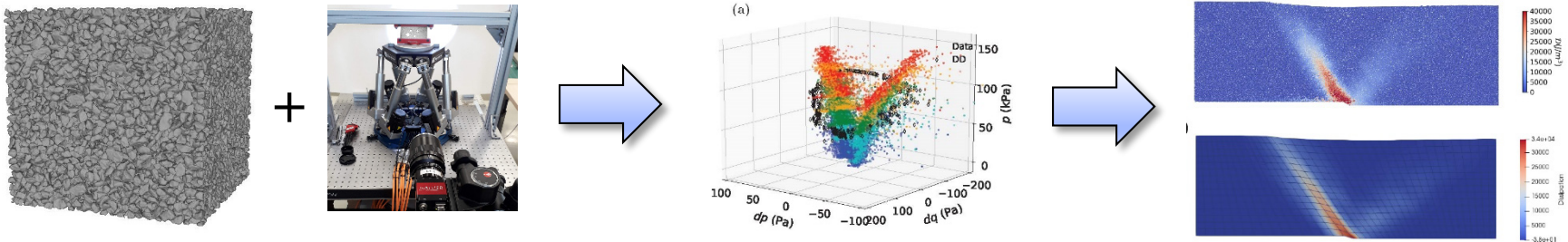


Top: LS-DEM simulation
Bottom: DD simulation

Evolution of surface settlement,
LS-DEM vs. DD simulations

Data mining, generation, upscaling

- DDMI can extract data from *full-field experimental microscopy* (TEM, SEM, DIC, EBSD...)
- DDMI generates *fundamental data without prior assumptions* on the form of constitutive response
- Data can be *mined* from lower-scale calculations, used in upper-scale calculations (*DD upscaling*)
- *DD sets forth new opportunities for synergism between experimental science and scientific computing, new multiscale analysis paradigms*

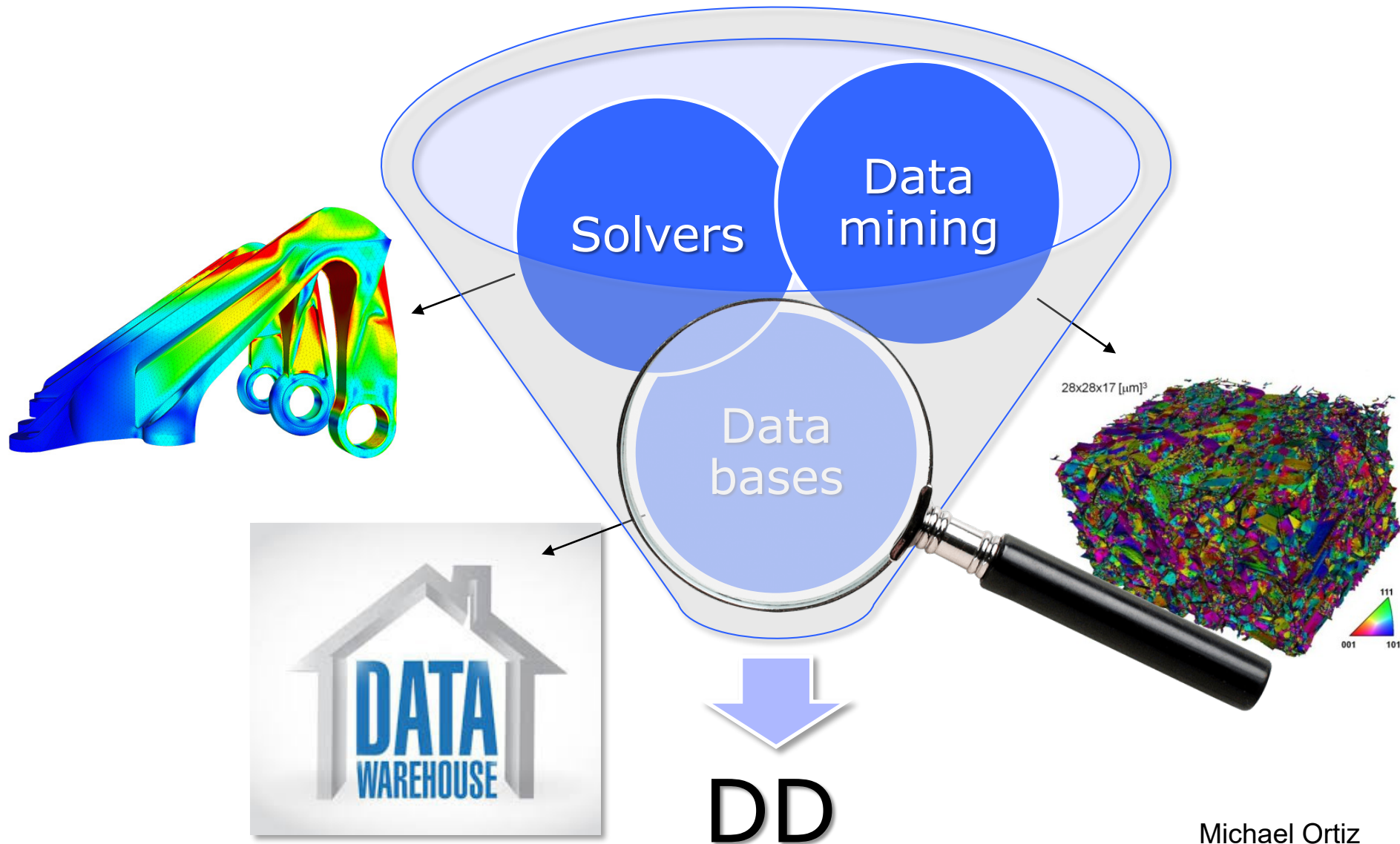


J. Rethore and A. Leygue, HAL Id: hal-01452494, Feb. 2017.

K. Karapiperis, L. Stainier, M. Ortiz, J.E. Andrade, *JMPS*, (2020) 104239.

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The (model-free) DD ecosystem...



Publically-editable data repositories

- Reliance on *fundamental data* (stress and strain only, no model-dependent data) makes *material data fungible*, mergeable, interchangeable...
- *Publically editable repositories* have proven exceptional capacity for *organic growth*...
- *Publicly-editable material data repository! (Wikimat)*
 - *Fundamental, model-independent data:*
 - *Stress-strain*
 - *Temperature-entropy*
 - *Gradients, rates*
 - *Tools/Scripts for interfacing with commercial FE packages*



WIKIPEDIA
The Free Encyclopedia

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Concluding remarks

- *Model-Free Data-Driven computing: The data, all the data, nothing but the data!*
- *The Model-Free Data-Driven ecosystem:*
 - Solver standardization (material independent)
 - Data standardization (model-independent data)
 - Lossless, set-oriented Machine Learning
 - Data identification from full-field microscopy
 - Data identification from multiscale analysis
 - Data repositories (fungible, publically editable)
- *Data-driven computing is likely to be a growth area in an increasingly data-rich world and to change the way in which data is mined, stored, exchanged, disseminated and utilized in science and in industry!*

Concluding remarks

Thank you!