Three-dimensional cohesive modeling of dynamic mixed-mode fracture

Gonzalo Ruiz\textsuperscript{1}, Anna Pandolfi\textsuperscript{2} and Michael Ortiz\textsuperscript{3,*}\textsuperscript{,†}

\textsuperscript{1}E. T. S. de Ingenieros de Caminos, Canales y Puertos, Universidad de Castilla-La Mancha, 13071 Ciudad Real, Spain

\textsuperscript{2}Dipartimento di Ingegneria Strutturale, Politecnico di Milano, 20133 Milano, Italy

\textsuperscript{3}Graduate Aeronautical Laboratories, California Institute of Technology, Pasadena, CA 91125, U.S.A.

SUMMARY

A cohesive formulation of fracture is taken as a basis for the simulation of processes of combined tension-shear damage and mixed-mode fracture in specimens subjected to dynamic loading. Our three-dimensional finite-element calculations account explicitly for crack nucleation, microcracking, the development of macroscopic cracks and inertia. In particular, a tension-shear damage coupling arises as a direct consequence of slanted microcrack formation in the process zone. We validate the model against the three-point-bend concrete beam experiments of Guo \textit{et al.} \textit{(International Journal of Solids and Structures} 1995; \textbf{32}(17/18):2951–2607), John \textit{(PhD Thesis}, Northwestern University, 1988), and John and Shah \textit{(Journal of Structural Engineering} 1990; \textbf{116}(3):585–602) in which a pre-crack is shifted from the central cross-section, leading to asymmetric loading conditions and the development of a mixed-mode process zone. The model accurately captures the experimentally observed fracture patterns and displacement fields, as well as crack paths and crack-tip velocities, as a function of pre-crack geometry and loading conditions. In particular, it correctly accounts for the competition between crack-growth and nucleation mechanisms. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: tension-shear damage coupling; mixed mode dynamic fracture; three-point-bend concrete beams; cohesive models; 3D finite element

1. INTRODUCTION

Brittle materials, such as concrete, mortar, rocks and ceramics, often develop complex fracture patterns when they are subjected to high rates of loading. The elastic heterogeneities and weak interfaces present in these materials on the microscale often result in the nucleation of microcracks, which may in turn coalesce into large structural cracks. Such cracks follow

\textsuperscript{*}Correspondence to: Michael Ortiz, Graduate Aeronautical Laboratories, California Institute of Technology, Firestone Flight Sciences, Mail Stop 105-50, Pasadena, CA 91125, U.S.A.

\textsuperscript{†}E-mail: ortiz@aero.caltech.edu

Contract/grant sponsor: Caltech’s ASCI/ASAP Center for the Simulation of the Dynamic Properties of Solids

Contract/grant sponsor: Dirección General de Enseñanza Superior, Ministerio de Educación y Cultura

Copyright © 2001 John Wiley & Sons, Ltd.
meandering paths and undergo frequent branching depending upon the loading and boundary conditions. Cracks may link up with free surfaces or with each other to form fragments. In concrete, the microstructural length scale is commensurate with the aggregate size and, thus, fracture processes often occur on a scale comparable to the geometrical dimensions of the structure. In addition to the energy required to fracture a material, other sources of energy dissipation may operate simultaneously, including plastic work, viscosity, and heat conduction. Finally, microinertia often plays a significant role in shaping the effective macroscopic behaviour of solids at high rates of deformation (e.g. References [1–5]).

Fracture mechanics specifically addresses the issue of whether a body under load will remain intact or whether a new free surface will form. However, classical fracture mechanics assumes a pre-existing dominant crack and thus foregoes the issue of nucleation. Additionally, the conditions for crack growth are typically expressed in terms of parameters characterizing the amplitude of autonomous near-tip fields, which restricts the scope of the theory. e.g. by requiring that the plastic or process zone be small relative to geometrical dimensions such as the crack or ligament sizes. For concrete, these conditions are rarely realized in practice. In classical dynamic fracture mechanics, the fracture criterion and crack-tip equation of motion must also account for the microinertia which accompanies the motion of the crack tip.

An alternative approach to fracture, which overcomes some of the aforementioned difficulties, is based on the use of cohesive models [6–35]. In these theories, fracture is regarded as a gradual process in which the separation of the incipient crack flanks is resisted by cohesive tractions. The relation between the cohesive tractions and the opening displacements is governed by a cohesive law. Cohesive models furnish a complete theory of fracture which is not limited by any consideration of material behaviour, finite kinematics, non-proportional loading, dynamics, or the geometry of the specimen.

In addition, cohesive theories fit naturally within the conventional framework of finite-element analysis, and have proved effective in the simulation of complex fracture processes including: the fragmentation of brittle materials [24, 36]; dynamic fracture and fragmentation of ductile materials [31, 37]; and the dynamic Brazilian test for concrete [32]. However, Camacho and Ortiz [24] have noted that the accurate description of fracture processes by means of cohesive elements requires the resolution of the characteristic cohesive length of the material. In some materials, such as ceramics and glass, this length may be exceedingly small. Thus calculations based on cohesive elements inevitably have a multi-scale character, in as much as the numerical model must resolve two disparate length scales commensurate with the macroscopic dimensions of the solid and the cohesive length of the material.

In this paper, we use cohesive theories of fracture to simulate processes of tension-shear damage and mixed-mode fracture in concrete specimens subjected to dynamic loading. We adopt a simple cohesive law proposed by Camacho and Ortiz [24] which accounts for tension-shear coupling through the introduction of an effective scalar opening displacement. The form of the effective opening displacement allows for different weights to be applied to the normal and tangential components of the opening displacement vector. This simple device also permits according the material critical normal and shear tractions bearing arbitrary ratios. This flexibility is particularly important in the case of concrete, where the aggregate interlocking mechanism may result in critical shear tractions greatly in excess of the tensional strength of the material. The cohesive behaviour of the material is assumed to be rigid, or perfectly coherent, up to the attainment of an effective traction, at which point the cohesive surface
begins to open. The cohesive law is rendered irreversible by the assumption of linear unloading to the origin.

In calculations, we allow for decohesion to occur along element boundaries only. Initially, all element boundaries are perfectly coherent and the elements are conforming in the usual sense of the displacement finite-element method. When the critical cohesive traction is attained at the interface between two volume elements, we proceed to insert a cohesive element at that location. Crack nucleation is thus accounted for and falls within the scope of the analysis. The cohesive element subsequently governs the opening of the cohesive surface. It is important to note that the cohesive size of concrete scales with the aggregate size. Since, as noted earlier, this cohesive length must be resolved numerically, the element size must be of the order of the aggregate size over the cohesive zone. Thus, the insertion of a new cohesive element may be regarded as the nucleation of one additional microcrack. These microcracks may subsequently coalesce among themselves, resulting in the nucleation and growth a structural crack.

Since the fracture surface is confined to inter-element boundaries, the structural cracks predicted by the analysis are necessarily rough. Cracks in concrete, as well as in materials which crack by intergranular fracture, exhibit considerable roughness on the scale of the aggregate or grain size. The surface roughness adds to the fracture area and, consequently, increases the specific fracture energy. Severe roughness may also toughen the material by the aggregate interlocking and crack bridging mechanisms. By the choice of an element size comparable to the aggregate size, the numerical model furnishes a simple description of the actual crack-surface roughness which is observed experimentally and the attendant effective toughening of the material.

It is also important to note that, owing to the presence of profuse microcracking in the vicinity of the crack tip, the effective fracture behaviour of the material may differ significantly from that which obtained by exercising the cohesive law directly. In particular, when a pre-crack is subjected to pure mode-II loading a number of slanted microcracks are inevitably formed ahead of the crack tip, roughly at 45° to the plane of the pre-crack. The subsequent behaviour of the crack is then sensitively dependent on the extent of normal compression, which tends to close the trailing microcracks and constrains them to undergo frictional sliding. Evidently, this type of behaviour is not ‘hardwired’ into the class of cohesive laws adopted here, but it is predicted by them.

Likewise, the effective dynamic behaviour of the material is predicted as an outcome of the calculations, instead of being built into the cohesive law as a rate-sensitivity effect [1, 38–47]. In particular, we assume the cohesive law to be rate independent. Cohesive theories, in addition to building a characteristic length into the material description, endow the material with an intrinsic time scale as well [24]. This intrinsic time scale permits the material to discriminate between slow and fast loading rates and, in materials such as concrete, ultimately allows for the accurate prediction of dynamic fracture properties [32].

We validate the predictions of the model against the experimental data of Guo et al. [1], and John and Shah [2, 3]. These tests were concerned with three-point-bend beam (TPB) specimens containing a pre-crack shifted from the central cross-section, leading to asymmetric loading conditions and resulting in mixed-mode fracture. Specimens were tested dynamically by means of a drop-weight device [1], or a modified Charpy tester [2, 3]. By increasing its offset, the pre-crack is subjected to increasingly mode-II fracture conditions, and the trajectory of the crack varies accordingly. In addition, the experimental data is indicative of a competition between two fracture mechanisms: the growth of the pre-crack; and the nucleation and
subsequent growth of a crack within the central cross-section of the specimen. The fracture behaviour of the specimen is also observed to be sensitively dependent on the rate of loading.

These and other features of the experimental set-up and the observed behaviour of the specimens furnish an exacting test of the validity of the model. Our calculations show that the model is indeed capable of closely capturing the observed fracture patterns, crack trajectories and crack-tip displacement fields corresponding to different experimental configurations. In particular, it correctly accounts for the relative stability regimes of the competing pre-crack growth vs crack nucleation mechanisms. The simulations also return accurate histories of crack extension and velocity, and the computed loads agree closely with the reported experimental load histories.

The organization of the paper is as follows. A brief outline of the cohesive model and its finite-element implementation employed in calculations is given in Section 2 for completeness and subsequent reference. In Section 3 we review the set-up of the experiments of Guo et al. [1], and John and Shah [2, 3] which we aim to simulate. In Section 4 we collect or estimate the material properties used in subsequent calculations, with particular attention given to the tension-shear coupling constant $\beta$. We devote Section 5 to an assessment of the accuracy afforded by the discretization used in calculations. Selected results of the calculations and comparisons with experimental data are presented in Section 6. Finally, a summary and some concluding remarks are collected in Section 7.

2. COHESIVE MODEL OF FRACTURE

For completeness and subsequent reference, in this section we summarize the main features of the cohesive law used in calculations. An extensive account of the theory and its finite-element implementation may be found elsewhere [24, 29].

A simple class of mixed-mode cohesive laws accounting for tension-shear coupling (see Camacho and Ortiz [24] and others [28, 29]) is obtained by the introduction of an effective opening displacement $\delta$, which assigns different weights to the normal $\delta_n$ and sliding $\delta_S$ opening displacements

$$\delta = \sqrt{\beta^2 \delta_S^2 + \delta_n^2}, \quad \delta_n = \delta \cdot n, \quad \delta_S = |\delta_S| = |\delta - \delta_n n|$$

Here $\delta_n$ and $\delta_S$ are the normal and tangential opening displacements across the cohesive surface, respectively. Assuming that the cohesive free-energy density depends on the opening displacements only through the effective opening displacement $\delta$, the cohesive law reduces to [24, 29]

$$t = \frac{1}{\delta} (\beta t_S + \delta_n n)$$

where

$$t = \sqrt{\beta^{-2} |t_S|^2 + t_n^2}$$

is an effective cohesive traction, to be related to $\delta$ by a reduced cohesive law, and $t_S$ and $t_n$ are the shear and the normal tractions, respectively. From this relation, we observe that the weighting coefficient $\beta$ defines the ratio between the shear and the normal critical tractions.
In brittle materials, this ratio may be estimated by imposing lateral confinement on specimens subjected to high-strain-rate axial compression [48, 49]. It also roughly defines the ratio of $K_{IIc}$ to $K_{Ic}$ of the material.

We assume the existence of a loading envelope defining a relation between $t$ and $\delta$ under the conditions of monotonic loading. We additionally assume that unloading is irreversible. A simple and convenient type of irreversible cohesive law is furnished by the linearly decreasing envelope shown in Figure 1, where $\sigma_c$ is the tensile strength and $\delta_c$ the critical opening displacement. Following Camacho and Ortiz [24] we assume linear unloading to the origin, Figure 1.

Upon closure, the cohesive surfaces are subjected to the contact unilateral constraint, including friction. We regard contact and friction as independent phenomena to be modeled outside the cohesive law. Friction may significantly increase the sliding resistance in closed cohesive surfaces. In particular, the presence of friction may result in a steady—or even increasing—frictional resistance while the normal cohesive strength simultaneously weakens.

Cohesive theories introduce a well-defined length scale into the material description and, in consequence, are sensitive to the size of the specimen (see, e.g. Reference [8]). The characteristic length of the material may be expressed as

$$\ell_c = \frac{EG_c}{f_{ts}^2}$$

where $G_c$ is the fracture energy and $f_{ts}$ the static tensile strength. Camacho and Ortiz [24] have noted that, when inertia is accounted for, cohesive models introduce a characteristic time as well. This characteristic or intrinsic time is

$$t_c = \frac{\rho c \delta_c}{2f_{ts}}$$

where $\rho$ is the mass density and $c$ the longitudinal wave speed. Owing to this intrinsic time scale, the material behaves differently when subjected to fast and slow loading rates. This sensitivity to the rate of loading confers cohesive models the ability to predict subtle features of the dynamic behaviour of brittle solids, such as crack-growth initiation times and
propagation speeds [37], and the dependence of the pattern of fracture and fragmentation on strain rate [31]. In addition, they account for the dynamic strength of brittle solids, i.e. the dependence of the dynamic strength on strain rate [32].

In calculations, we allow for decohesion to occur along element boundaries only. Initially, all element boundaries are perfectly coherent and the elements are conforming in the usual sense of the displacement finite element method. When the critical cohesive traction is attained at the interface between two volume elements, we proceed to insert a cohesive element at that location. The cohesive element subsequently governs the opening of the cohesive surface. Details of the finite-element implementation may be found elsewhere [24, 29].

3. EXPERIMENTAL SET-UP

We specifically aim to simulate the experiments reported by Guo et al. [1], and John and Shah [2, 3]. Both sets of tests were carried out on prismatic plain concrete beams loaded in a three-point bend configuration. The specimens were notched at an offset with respect to their central cross-section to induce mixed-mode crack growth.

Guo et al. [1] used a drop-weight tester with a 10-kg impactor. In order to provide an averaged displacement history curve at the load point, the concrete beam was instrumented at the contact point with a dynamic load cell, plus two non-contact displacement gauges at the load point on both sides. The crack extension and velocity throughout the tests were measured by two different methods. On one side of the beam, five strain gauges were glued along the expected crack path; on the other side a moiré diffraction grating of 600 lines/mm spanning the area was transferred to the previously polished surface of the beam along the expected crack path. Upon contact with a push rod, the drop weight triggered several flashes and cameras which recorded the horizontal and vertical moiré patterns, from which the field of displacements around the crack—and the crack pattern itself—could be obtained. Details on this experimental procedure can be found in several papers by Kobayashi and coworkers [50–52], or in References [53, 54].

Gopalaratnam et al. [55] instrumented a Tinius Olsen 64 Charpy tester and adapted it to a three-point bending mixed-mode test on concrete specimens. Here we simulate John and Shah’s experiments [2, 3] performed with this apparatus. John and Shah measured the history of the load transmitted by the 27.3-kg tup of the pendulum, as well as the load exerted by the anvil supporting the specimen. Several strain gauges glued to the specimen allowed measurement of the strain rates. The crack advance and velocity were not recorded in these experiments. Nevertheless, the trajectories of the cracks through the beams could be recovered by the post-mortem examination of the specimens.

The geometry and dimensions of the prismatic beams used in both experimental programmes are depicted in Figure 2. Guo et al. extended the initial 19-mm machined notch by approximately 13 mm by stable symmetric loading. It should be noted that the relative offset, $\gamma = 2 \text{ offset}/L$, of the notch with respect to the central cross-section was systematically varied in John and Shah’s beams, with values: $\gamma = 0$, 0.5, 0.7, 0.72, 0.77 and 0.875.

In order to reduce the inertial effects on the measured load and to minimize the influence of the contact surface inhomogeneities on the test results, both Guo et al. [1] and John and Shah [2, 3] inserted a rubber pad between the impactor and the specimens. Guo et al. [1] recorded and reported the entire displacement history for the specimen point under the
impactor (Figure 3). The load history, also reported in Reference [1], provides a convenient basis for the validation of numerical results.

John and Shah’s tests are characterized by the strain rate, measured by a gauge located at the central cross-section of an unnotched beam, on the bottom side [2, 3, 55]. The strain rate is in turn related to the height from which the tup is dropped. After a transient effect due to the rubber pad, the strain rate reaches an ostensibly constant value. In calculations, we prescribe a constant velocity at the point of impact chosen so as to match the reported strain rate, $\dot{\varepsilon}_N$, imposing an uniform velocity at the impact point, $\dot{\varepsilon}$. Taking into account the geometry of the beam, the two values are related by the formula

$$\dot{\varepsilon} = \frac{L^2}{6D} \dot{\varepsilon}_N$$

where $L$ and $D$ are the beam span and depth. For the reported $\dot{\varepsilon}_N = 0.3/s$, (6) gives $\dot{\varepsilon} = 0.03 \text{ m/s}$. 

Figure 2. Geometry and dimensions of the beams tested by;
(a) Guo et al. [1]; and (b) John and Shah [2, 3].

Figure 3. Prescribed displacements under the load point, based on data by by Guo et al. [1].
4. MATERIAL CHARACTERIZATION

Table I summarizes the main properties of the concrete used in the experimental programmes of Guo et al. [1], and John and Shah [2, 3]. The only property measured by Guo et al. by means of an independent test was the compressive strength. The remaining material properties were identified by Guo et al. using inverse numerical analysis techniques. The values of the parameters were calibrated on the basis of two-dimensional dynamic finite-element analyses carried out using a cohesive model of fracture in conjunction with a prescribed crack path [1]. Specifically, Guo et al. used the load-line displacement and the horizontal and vertical crack opening displacement records, corresponding to the dynamic drop-weight tests to calibrate a linear cohesive law. By way of contrast, the material properties supplied by John and Shah were measured experimentally, with the sole exception of the tensile strength. We have estimated the tensile strength following the recommendations set forth in the CEB-FIP Model Code [56]. The shear critical traction is not documented in the original works of Guo et al. [1] or John and Shah [2, 3].

The material is assumed to obey the linear irreversible cohesive law shown in Figure 1, with the tensile strength taken as the cohesive strength of the material. In addition, in the calculations of interest here, and for the materials under consideration, the bulk behaviour of concrete can be idealized as elastic to a good approximation, (cf., e.g. Reference [57, Section7.1.6]). Indeed, the specimen is dominated by tensile cracking and the compressive strength of the material is not reached. As a slight and probably inconsequential improvement, in our calculations we assume that the material obeys $J_2$-plasticity following the attainment of its compressive strength. Details of the specific implementation of $J_2$-plasticity employed in calculations may be found elsewhere [58].

It should be carefully noted that, under the conditions envisioned in this study, and more generally when dealing with the fracture of concrete, conventional concepts from linear–elastic fracture mechanics must be applied with great caution or not at all. Thus, computing the characteristic length from (4) using the properties listed in Table I, we obtain 400 and 100 mm for the materials of Guo et al. and John and Shah, respectively. This characteristic length is larger than the depth of the beams, 95.25 and 76.2 mm, respectively. The condition of the specimens simulated here corresponds therefore to a ‘fully yielded’ situation and linear–elastic fracture mechanics, which fundamentally rests on the small-scale yielding condition, does not apply. By way of contrast, cohesive theories of fracture are not so constrained, and can be applied to situations such as the ones envisioned here, in which the size of the process zone is comparable to the ligament length or any other limiting geometrical dimension. Further discussions of this and related issues may be found in References [21, 57, 59].

<table>
<thead>
<tr>
<th>Table I. Concrete parameters.</th>
<th>( \text{Guo et al. [1]} )</th>
<th>( \text{John and Shah [2, 3]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum aggregate size (mm)</td>
<td>6.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Elastic modulus (GPa)</td>
<td>32.3</td>
<td>29.0</td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>33.8</td>
<td>39.7</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>3.1</td>
<td>3.0</td>
</tr>
<tr>
<td>Fracture energy (N/m)</td>
<td>120</td>
<td>31.1</td>
</tr>
</tbody>
</table>
The estimation of the tension-shear coupling parameter $\beta$, Equation (1), which defines the ratio between the shear and the normal critical traction for the cohesive model is of particular concern for concrete and requires some care. We have already remarked that $\beta$ roughly reflects the ratio of mode-II to mode-I fracture toughness. There is experimental evidence [60–69] that suggests that the intrinsic fracture toughness of concrete, i.e. the critical stress intensity factor required to advance a semi-infinite crack within its plane in the absence of kinking, is significantly larger in pure mode-II than in pure mode-I owing to the interlocking of aggregate particles, which suggests a large value of $\beta$. Indeed, Swartz et al. [60] calculated the ratio $K_{IIc}/K_{Ic}$ by analytical and numerical simulations of several mixed-mode tests. Their estimated values range between 3 and 6, depending on the type of specimen and on the underlying evaluation method.

We provide next a quantitative estimation of the $\beta$ from the mixed-mode experiments performed by Nooru-Mohamed and van Mier [63, 65]. Nooru-Mohamed and van Mier tested double-edge notched specimens subject to mixed-mode load (Figure 4(a)). Three different types of concrete were used, with maximum aggregate size equal to 2, 12 and 16 mm, respectively. The experimental set-up was designed to transmit independently the axial and the shear loads, following predetermined mixed-mode load paths, as sketched in Figure 4(b). We focus our attention on a particular load path, in which an increasing axial load was applied up to the point when a prescribed average crack width was obtained, followed by the complete unloading of the specimen (Figure 5(a)). An increasing shear load with displacement control was subsequently applied, keeping the axial load to zero and allowing for dilatancy of the crack, up to complete failure of the specimen (Figure 5(b)).

The first phase of the axial loading process generates a straight crack joining the two notches of the specimen. The material at the crack surfaces is therefore decohered and softened by tension, but its strength is not completely eliminated. During the application of the shear stress, the crack opens in pure mode-II, until the slip opening is too large and some dilatancy occurs (Figure 5(c)). Figure 6 shows the crack patterns obtained by this loading process for three different values of the mode-I maximum crack opening ($\delta = 50, 100$ and $150 \mu m$) on $200 \times 200 \times 50$ mm specimens of the 12-mm maximum aggregate size concrete. It should be noted that throughout the test the crack is subjected to pure-mode loading, first in mode-I and later in mode-II.

This load path has a straightforward interpretation within the context of the irreversible cohesive law shown in Figure 5(d). Thus, after the applied traction reaches the tensile strength
Figure 5. Load path followed to determine $\beta$: (a) axial loading and unloading; followed by; (b) loading in shear up to failure; and (c) history of the displacements $\delta$ and $\delta_s$; while (d) shows the load path deduced from the cohesive law.

Figure 6. Crack patterns corresponding to experiments by Nooru-Mohamed and van Mier [63, 65].

Figure 7. Upward view of some of the meshes used to simulate: (a) experiments of Guo et al.; and (b) John and Shah tests ($\gamma = 0.5$).
of the specimen, a cohesive crack opens up to the prescribed opening displacement, thereby reducing the normal bearing capacity of the specimen to $P_n$. At the onset of unloading, the average normal traction is $t_n = P_n / A$, $A$ being the area of the unbroken ligaments. The specimen is then reloaded in shear up to a load $P_s$, corresponding to a shear traction $t_s = P_s / A$, and brought to failure along the descending cohesive envelop. The axial load $P_n$ at the maximum displacement and the shear peak load $P_s$ correspond to the same point in the ‘effective’ cohesive envelop, Figure 5(d). Thus, according to (3), the value of $\beta$ is

$$\beta = \frac{P_s}{P_n}$$

(7)

Applying (7) to the experimental results of Nooru-Mohamed and van Mier [63, 65], we obtain $\beta = 4$, 6.6 and 6 for the 2, 12 and 16-mm maximum aggregate size concretes, respectively. These results provide an indication of the range of the tension-shear coupling parameter $\beta$ for concrete. These values are consistent with the aforementioned results obtained by Swartz et al. [60] for the ratio $K_{IIc}/K_{lc}$. Based on these considerations, in our simulations we set $\beta = 5$.

5. COMPUTATIONAL MESH AND CONVERGENCE TESTS

Figure 7(a) shows the mesh used in our simulations of Guo et al. experiments, whereas Figure 7(b) shows one of the meshes used in the simulations of John and Shah’s tests, corresponding to a relative offset $\gamma = 0.5$. The boundary surfaces and the interior of the specimens are meshed automatically by an advancing front method [70] using 10-node quadratic tetrahedra. In order to cut down on the size of the calculations, the finite-element meshes are designed so as to be fine and nearly uniform on and in the vicinity of the expected crack paths. The element size equals half the maximum aggregate size in the fine mesh region and gradually coarsens away from that region up to the maximum aggregate size. It should be carefully noted that coarse mesh has a somewhat lower numerical toughness than the fine mesh [24] and, consequently, does not introduce a barrier for the propagation of the path. Thus, the presence of a fine mesh region and the mesh gradation used in the calculations does not bias the path of the crack.

It bears emphasis that the maximum mesh size is 1/25 of the characteristic cohesive length (4) of the material and may, therefore, be expected to yield objective and mesh-size insensitive results [24]. In order to verify this point, we have compared solutions obtained from two differently refined meshes. The tests correspond to John and Shah’s specimens for $\gamma = 0$ and $\dot{\varepsilon}_N = 1/s$. The results of the two tests are displayed in Figure 8. The reaction histories at the supports differ negligibly up to the third peak. The waves in the curves are due to the oscillation of the beam and are also in keeping with the experiments. The difference in cohesive energy consumption in the two meshes is also very small, about 1 per cent. Figure 8 compares the front view of the crack advance for the two meshes at two different stages of the analysis, 250 and 600 $\mu$s after the impact of the tup. Also shown in the figure are level contours of damage, defined as the fraction of expended fracture energy to total fracture energy per unit surface, or critical energy release rate. The crack advance and the damage distribution comparison provide a further indication of the good resolution provided by the coarse mesh.
In order to verify the accuracy of the coarse mesh in the presence of crack nucleation, we have carried out additional tests without a pre-notch. Figure 9 shows the numerical results for both the fine and coarse meshes. As in the pre-notch case, we again find that the load curves match up to the peak load. However, the cohesive energy consumption is significantly larger for the fine mesh. This discrepancy is also apparent in the level contours in damage,
which reveal a broader microcracked zone for the fine mesh; and in the crack profile, which is sharp in the case of the coarse mesh and comparatively rougher and unsteady for the fine mesh. The larger extent of microcracking which accompanies nucleation in the fine mesh in turn accounts for the higher level of energy dissipation.
These observations provide an illustration of our incomplete understanding of issues of numerical convergence of cohesive finite-element models, specially in the presence of nucleation, branching and fragmentation. Thus, while it appears quite clear that finite-element solutions begin to exhibit strong convergence once the cohesive length is resolved (e.g. Reference [24]), the situation is far more uncertain for more complex fracture processes. It is not entirely clear at this time whether the increase of microcracking as the mesh is refined in the tests just discussed is a numerical artefact or reflects the actual physics of the crack nucleation process in concrete. Or, in this latter case, how and in what sense the finite-element solution converges in the limit of infinite refinement. For instance, numerical experiments [32] seem to indicate that, while the local details of the microcracking pattern are vastly non-unique, certain microscopic measures, such as energy dissipation, are quite reproducible and appear to converge properly under mesh refinement. These issues clearly suggest themselves as worthwhile subjects for future research.

6. SIMULATION RESULTS

Selected results of the calculations and comparisons with experimental data are presented in this section.

6.1. Crack pattern

Figure 10 shows four snapshots of the simulation of one of John and Shah’s Charpy tests corresponding to a relative offset $\gamma = 0.5$ [2, 3]. The snapshots record the evolution of the crack pattern. Specifically, the figure shows the intersection of the cohesive elements with the surfaces of the specimen. As may be seen from the figure, the dominant fracture mode for this geometry consists of the extension of the pre-notch towards the point of application of the load. This structural crack is accompanied by a certain amount of distributed microcracking and profuse branching, as expected in concrete. However, it should be carefully noted that some of the microcracks and crack branches are partially failed and should not be construed as completely formed free surfaces.

Simultaneously with the growth of the main crack, a diffuse microcracking zone forms around the tensile fiber of the central cross-section. There is also evidence of distributed damage at the point of contact with the impactor. These distributed damage zones act as additional energy sinks. The tensile microcrack zone remains diffuse throughout the simulation and fails to develop into a structural crack. Thus, Figure 10 suggests that two failure modes are in competition: the extension of the pre-notch and the nucleation of a new crack within the loading plane. For the particular geometry shown in Figure 10, the pre-notch extension mode clearly wins out at the expense of the nucleation mode.

Figure 11 compares the experimentally and predicted crack paths in two cases: the first for the geometry of Guo et al., Figure 11(a), and the second for John and Shah, Figure 11(b). The results of the calculations are shown as level contours of cohesive damage. Superimposed on the figures are the experimental observed bounds of the crack trajectories (thick solid line). In Figure 11(b), the straight line emanating from the notch tip is a linear–elastic fracture mechanics prediction for the crack-growth direction reported by John and Shah [2, 3]. As is evident from these figures, the predicted crack paths are in excellent agreement with the experimentally observed crack trajectories.
Figure 10. Snapshots of the simulation of John and Shah’s Charpy tests on concrete specimens [2, 3], showing the evolution of the crack pattern.

Figure 11. Computed damage distribution (shaded mesh) compared to the experimental crack pattern (thick solid lines) by: (a) Guo et al. Hawkins [1]; and (b) John and Shah [2, 3].
6.2. Influence of the notch offset on the crack pattern and failure mode

As already mentioned, the TPB specimens tested by John and Shah had a notch offset from the center of the beam [2, 3]. The notch offset was primarily intended as a means of varying the mode-II/mode-I ratio. Thus, for instance, an increase in the notch offset was observed to result in a corresponding increase in the kinking angle for crack-growth initiation, which is indicative of a higher mode mixity. Interestingly, as already mentioned at high offsets John and Shah observed a change in failure mode from pre-notch extension to crack nucleation within the plane of the load. This exchange of stability was observed to occur at a critical offset of roughly 0.7.

Figure 12 displays computed crack patterns for relative offsets $\gamma = 0, 0.5, 0.6$ and 1, respectively, and thus illustrates the dependence of the predicted failure mode on notch offset. The calculations are in good qualitative agreement with the experimental observations of John and Shah [2, 3], although the critical offset for the exchange of stability is somewhat underpredicted at about 0.6.

However, it should be noted that the critical offset depends sensitively on the tension-shear coupling parameter $\beta$. Thus, a high (low) value of $\beta$ describes a material which is stronger in shear (compression) than in compression (shear). Correspondingly, a high value of $\beta$ inhibits the development of mode-II cracks and thus promotes the nucleation of a mode-I crack within the central cross section. Contrariwise, a low value of $\beta$ favours the extension of the notch. This dependence is shown in Figure 13, which collects the crack patterns obtained in simulations of John and Shah’s test [2, 3] with a fixed relative offset $\gamma = 0.5$ and for values of $\beta = 0.1, 1, 5$ and 10. In particular, a transition in failure mode from pre-notch extension to crack nucleation is observed between values of $\beta = 5$ and 10. It follows from this analysis that the critical offset value $\gamma = 0.7$ observed by John and Shah may be matched exactly by suitably increasing the value of $\beta$ above the value $\beta = 5$ used in calculations, but this enhancement will not be pursued here.
Figure 13. Numerical simulations of the three-point-bend experiments of John and Shah [2, 3]. Effect of the tension-shear coupling constant $\beta$ on crack pattern failure mode.

6.3. Displacements field around the crack

Guo et al. [1] used a Moiré diffraction grating in order to measure the specimen surface displacement field around the crack tip. These measurements provide a wealth of data regarding the strain and stress fields over the crack-tip area. Figure 14 shows a typical sequence of Moiré patterns associated with horizontal and vertical displacements in the vicinity of the main crack [1]. Regrettably, in the original paper the time elapsed between two consecutive snapshots and the displacement increment between contours is not reported. This limitation notwithstanding, the measured Moiré interferometry patterns permit a useful qualitative comparison with the numerical predictions.

Figure 15 shows synthetic Moiré interferometry patterns constructed from the results of the calculations. We choose a time delay between consecutive images of 100 $\mu$s. Solid (dashed) lines denote positive (negative) displacements. A careful comparison between Figure 14 and Figures 15(a) and 15(b) reveals an excellent overall agreement between simulation and observation. This general agreement notwithstanding, it is interesting to note that the measured patterns are smoother than the computed ones in the immediate vicinity of the crack tip and show no signs of microcracking. Following Post [53], a plausible explanation is that the Moiré grating is strong enough to remain intact away from the main crack and thus covers the underlying microcracks or inhibits their formation altogether. It is also interesting to note in Figure 15(c), which shows the level contours of normal out-of-plane displacement on the surface of the specimen, that the heavily microcracked process zone at the tip of the crack bulges out of the specimen. This illustrates the strong dilatancy which the model predicts to occur simultaneously with intense distributed damage.

6.4. Crack extension and velocity

The sequence of Moiré patterns obtained by Guo et al. in their experiments allowed them to determine the main crack-tip trajectory, and by numerical differentiation, the crack-tip velocity...
history, Figure 16. The TPB specimens were also instrumented with five strain gauges located over the expected path of the crack on the side of the specimen not covered by the Moiré grating. The agreement between both sets of data attests to the accuracy of the measurements.

Figure 16 also superposes the predicted crack-length and crack-tip velocity histories on the experimental data. The onset of crack growth coincides with the arrival of tensile waves to the pre-notch tip roughly 320 μs after impact. The crack quickly settles down to a nearly constant velocity of approximately 170 m/s during a time interval of 200 μs. At the end of this interval, the crack slows down and arrests owing to the arrival of relief waves and the interaction between the crack tip and the damaged zone under the impactor. The predicted crack-growth initiation time is consistent with observation. The agreement between the calculated and measured crack trajectories is excellent. The computed crack-tip velocity history exhibits two crack-speed peaks separated by a somewhat slower regime. This crack slow-down occurs simultaneously with a burst in microcrack activity around the crack tip.

6.5. Load history curves

Figure 17 compares the experimental and computed load histories corresponding to tests of Guo et al. [1]. It should be carefully noted that the numerical model assumes a mathematically sharp and completely formed pre-notch, whereas in the experiments the pre-notch was introduced by stable symmetric loading. This procedure is likely to result in a pre-crack which is bridged by intact ligaments and, consequently, has some residual strength. Owing to this
Figure 15. Sequence of computed contours of (a) horizontal displacement; (b) vertical displacement; and (c) out-of-plane displacement.

Within this context, the numerical and measured load histories are not comparable during the early stages of the test, and are shown as dashed lines in Figure 17. Beyond this point, the experimental and numerical load histories exhibit two peak loads, Figure 17. The time of occurrence and the intensity of the first peak is well predicted by the model. The subsequent agreement between the experimental and calculated load histories is poor. In this regard it should be noted that the first peak occurs when the amount of
crack extension is small, and the structure of the process zone is largely determined by the cohesive strength of the material. By contrast, the second peak occurs after substantial crack growth has taken place, and the entire cohesive law has come into play. It is thus likely that the simple linear cohesive envelop assumed in the calculations is not sufficient to match the experimental observations accurately, and that a better agreement requires the use of more elaborate cohesive laws.

7. SUMMARY AND CONCLUSIONS

We have taken a cohesive formulation of fracture as a basis for the simulation of processes of combined tension-shear damage and mixed-mode fracture in solids subjected to dynamic
loading. The cohesive model accounts for key fracture properties such as the cohesive strength and the fracture energy, and, in conjunction with inertia, introduces characteristic length and time scales into the material description. The particular model proposed by Camacho and Ortiz [24], and subsequent extensions thereof [29–32], also accounts for tension-shear coupling and permits according the shear and tensile strengths independent values bearing arbitrary ratios. The cohesive law is assumed to be rate-insensitive and, therefore, all rate effects predicted by the theory are a consequence of the interplay between inertia and fracture [24, 31; 32]. This is in contrast to most past simulations of the dynamic fracture of concrete, which directly model the tensile strength as an increasing function of strain rate [1, 38–47].

In calculations, the fracture surface is confined to inter-element boundaries and, consequently, the structural cracks predicted by the analysis are necessarily rough. However, in simulations of concrete this numerical roughness can be made to correspond to the physical roughness by the simple device of choosing the element size to be comparable to the aggregate size. This choice of element size also serves to resolve the cohesive zone size, which in concrete is a small multiple of the aggregate size. The resulting simulations thus have a multiscale character, in as much as the discretization is charged with resolving both a micromechanical scale—the cohesive lengthscale—and the structural dimension.

We have validated the predictions of the model against the experimental data of Guo et al. [1], and John and Shah [2, 3]. These tests concerned three-point-bend beam (TPB) specimens containing a pre-crack shifted from the central cross section and subject to dynamic loading. The presence of a pre-notch in the specimen leads to asymmetric loading conditions and results in mixed-mode fracture. Our calculations show that the cohesive model is indeed capable of closely capturing the observed fracture patterns, crack trajectories and crack-tip displacement fields corresponding to different experimental configurations. The simulations also return accurate histories of crack extension and velocity.

The relative ease with which cohesive theories of fracture are capable of describing complex crack patterns, often involving profuse microcracking, and crack trajectories is particularly noteworthy. This ease is in contrast with the complexities inherent to the tracking mathematically sharp three-dimensional cracks by conventional linear–elastic fracture mechanics criteria [71–74]. The classical fracture mechanics paradigm becomes intractable in situations involving complex patterns of interacting cracks, e.g. in zones of distributed microcracking or in the face of fragmentation processes; and, for all practical purposes, it plainly ceases to apply when such complex fracture patterns occur in combination with inelasticity, finite deformations, dynamics, free surfaces and interfaces, and other similar complicating circumstances.

Cohesive theories also facilitate the treatment of crack nucleation and coalescence. Our calculations demonstrate the ability of the theory to predict the formation of diffuse microcracking zones and the nucleation of structural cracks, e.g. by microcrack coalescence. These fracture mechanisms are clearly observed in the experiments of John and Shah [2, 3]. The competition between crack growth and crack nucleation, and the exchange of stability between the two failure modes at a critical pre-notch offset, are well predicted by the theory.

Finally, we conclude by suggesting several subjects for further research. Thus, while in our calculations the computed load histories agree closely with experiment up to the first peak, the subsequent agreement is amenable to improvement. This most likely requires the formulation of more elaborate cohesive laws, possibly containing a larger material parameter set. In addition, while our numerical tests show strong convergence of the finite element
solutions in the presence of a sharp pre-notch, the convergence properties of the solutions when crack nucleation is involved remain to be fully ascertained.

ACKNOWLEDGEMENTS

The support of Caltech’s ASCI/ASAP Center for the Simulation of the Dynamic Properties of Solids is gratefully acknowledged. Gonzalo Ruiz gratefully acknowledges the financial support for his stay at the California Institute of Technology provided by the Dirección General de Enseñanza Superior, Ministerio de Educación y Cultura, Spain. We are indebted to Santiago Lombeyda, Research Scientist at the Center for Advanced Computing Research, Caltech, for rendering the simulation results.

REFERENCES