Finite element simulation of dynamic fracture and fragmentation of glass rods

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Received 7 September 1998

Abstract

The aim of this communication is to provide further illustration of the feasibility of simulating fragmentation explicitly, crack by crack. Cracks are allowed to form and propagate along element boundaries in accordance with a tensile-shear cohesive-law model. No topological restrictions are imposed on the cracks, which may nucleate at the surface or in the interior, branch, and link up to form fragments. As the fragments scatter, the complex collisions which they undergo and the attendant frictional interactions are also resolved explicitly by recourse to a contact algorithm. We present a new model of radial cracking which permits the calculation of normal impact to proceed in an asymmetric mode, without artificially constraining fragment rotation within meridional planes. The scope and versatility of the approach is demonstrated by simulating the propagation of failure waves in glass rods subjected to impact. Key aspects of the observational evidence, such as the failure wave speeds, are correctly predicted by the simulations. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Failure waves; Fragmentation; Finite element modeling

1. Introduction

The response of brittle materials to impact loading has been extensively investigated experimentally [1,2,7,17,23–25,28–31,33,78,83–45,47,48,73–75,84]. In particular, glass plate and rod impact experiments [4–6,8,10,11,26,27,41,67], have revealed so-called ‘failure waves’ in the form of propagating phase boundaries which separate an essentially intact material ahead of the wave front from a comminuted material behind the wave front. Failure wave propagation velocities, which are not a material constant but rather depend on loading conditions and specimen geometry, have been reported to exceed the theoretical maximum crack propagation speed. For instance in bars, Brar et al. report failure wave speeds which range from 2.3 to 5.2 mm/μs, increasing with increasing impact velocity. In plates, Brar et al. report failure wave speeds around 2 mm/μs [9]. The failure is further observed to be explosive in nature, leading to radial bursting in bars and a sharp increase in mean stress in plates. These observations have elicited considerable speculation and the theoretical understanding and numerical modeling of failure waves remains active area of research at present.

Past models of impact damage are for the most part based on continuum damage theories in which the net effect of fracture is idealized as a degradation of the elasticity of the material [20,21,22,27,39,54,56,57,59–61,60–70,72,82]. In addition, fragmentation has often been modelled by recourse to global energy balance concepts [34]. However, continuum theories of fracture and fragmentation

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suffer from obvious shortcomings. Thus, the discrete nature of cracks is lost in these theories. In homogenizing a cracked solid, sweeping assumptions must necessarily be made regarding the distribution and geometry of the cracks, which at best are described by a few state variables, and their interactions. The determination of the effective properties of a cracked solid under dynamic conditions presents additional difficulties stemming from the finite speed at which signals propagate between cracks [32]. However, perhaps the most fundamental objection to continuum theories is that the failure of a brittle specimen is frequently governed by the growth of a few single dominant cracks, a situation which is not amenable to homogenization. A related issue concerns the lack of a well-defined fracture energy, in the sense of an energy of separation per unit area, in continuum damage models. In finite element simulations this results in spurious mesh dependencies and generally in a lack of convergence as the mesh size is reduced to zero. In particular, continuum damage models predict a vanishing apparent fracture energy in the limit of a zero mesh size, i.e., it takes zero energy to propagate a crack.

Camacho and Ortiz [12,55], in two dimensions, and Ortiz et al. [63,64], in three dimensions, have established the feasibility of: (i) accounting explicitly for individual cracks as they nucleate, propagate, branch and possibly link up to form fragments; and (ii) simulating explicitly the granular flow which ensues following widespread fragmentation. In this approach, cracks are allowed to form and propagate along element boundaries in accordance with a tension-shear cohesive-law model [12,14,22,36,51–53,58,62,66,79–81,83,85–88]. Clearly, it is incumbent upon the mesh to provide a rich enough set of possible fracture paths, an issue which may be addressed within the framework of adaptive meshing. New surface may then be adaptively created as required by the cohesive model by duplicating nodes along previously coherent element boundaries, and the nodes are subsequently released in accordance with the cohesive law. These simple rules permit the simulation of strikingly complex crack patterns including branching, surface cracking and fragmentation [12,55,63], such as arise in the glass rod and plate experiments alluded to above.

If the extent of fragmentation is sufficiently severe, the comminuted phase flows as a granular material. Under these conditions, the multiple collisions and frictional interactions between the fragments have to be monitored efficiently. In particular, the large number of simultaneous contacts requires the contact search algorithm to be optimized for computational efficiency. Algorithmically demanding contact situations, such as those involving sharp corners, occur with some frequency and need to be resolved effectively. Robust contact algorithms capable of dealing with complex contact situations involving simultaneous interactions between several bodies with corners have been developed by Kane et al. [40]. They specifically address contact geometries for which both normals and gap functions are undefined, which precludes the application of conventional contact algorithms. Nonsmooth contact algorithms such as developed by Kane et al. [40] make it possible to resolve the complex collision sequences undergone by tightly packed angular fragments before scattering, thereby facilitating the explicit simulation of fragmentation debris flow.

While the methodology developed here applies equally in two and three dimensions, the scale of the calculations can be reduced considerably in problems possessing axial symmetry, e.g., normal impact by a solid of revolution. If the calculations are restricted to a meridional plane, thereby effecting a much desirable reduction in dimensionality, radial cracks can no longer be monitored explicitly and have to be modelled in a continuum sense. The computational pay-off in these cases is considerable enough that, by way of compromise, a hybrid formulation may be adopted in which conical and lateral cracks are modelled discretely while radial cracks are modelled continuously. Camacho and Ortiz [12] resorted to a damage model to simulate radial cracking. Here we present an alternative approach in which the radial cracks are simulated using a cohesive model. This approach has the advantage that the same fracture model and fracture parameters are used to simulate all cracks. The net effect of the radial cohesive cracks is to allow for a continuous transition between axisymmetric states of stress in regions devoid of radial cracks to plane stress states in regions which are finely fractured in the circumferential direction. In particular, fragments in these latter regions are free to rotate within meridional planes, as opposed to being constrained to move up and down as rings as would be the case if radial cracking were not taken into account.

The main objective of this communication is to showcase the predictive character of the numerical methodology just outlined for a particular validation example, namely the propagation of failure waves in glass rods. Indeed, the calculations shown below correctly capture the development and propagation of a sharp failure wave, its propagation speed and the bursting of the comminuted material following the passage of the failure wave.
2. Finite element model

We begin with a brief description of the computational framework on which we have based our calculations. Dynamic impact is ideally suited to simulation within an explicit dynamics framework. In addition, the distortion and rotation of fragments necessitates consideration of large deformation kinematics. We enforce dynamic equilibrium weakly by recourse to the principle of virtual work. Upon finite element discretization, the linear momentum balance equation becomes

$$M\ddot{x} + F_{\text{int}}(x, \dot{x}) = F_{\text{ext}},$$  \hspace{1cm} (1)$$

where \(x\) is the array of nodal coordinates, \(M\) the mass matrix, \(F_{\text{ext}}\) the external force array, and \(F_{\text{int}}\) is the internal force array corresponding to the current state of stress. All calculations proceed incrementally and the solution is sampled at discrete times \(t_0, \ldots, t_n, t_{n+1} = t_n + \Delta t, \ldots\). We use the second-order accurate central difference algorithm to discretize (1) in time [37, 38, 3]. Fragmentation events such as contemplated here set in motion a complex sequence of contact events between large numbers of deformable bodies. These complex contact situations must be effectively resolved during simulation. In the simulations presented here, we follow all the collisions between fragment and their frictional interactions by an extension of the contact algorithm of Taylor and Flanagan [77].

During impact events, materials frequently experience considerable volumetric and shear deformations. The volumetric or dilatational response is presumed to be governed by a Mie-Grüneisen equation of state. In addition, a small artificial bulk viscosity is introduced to prevent high velocity gradients from collapsing elements and to quiet down ringing [77]. For the deviatoric response we adopt a standard formulation of finite deformation plasticity based on a multiplicative decomposition of the deformation gradient into elastic and plastic components. We employ the fully implicit algorithm of Cuitiño and Ortiz [19] for performing the constitutive updates. In a typical impact penetration event, very high strain rates in excess of \(10^5 \text{ s}^{-1}\) may be attained. Under these conditions, a power viscosity law with constant rate sensitivity \(m\) is not adequate. Indeed, the experimental stress-strain rate curves for ceramics [44, 76] and metals [47, 18, 89] exhibit a transition at strain rates of the order of \(10^3-10^6 \text{ s}^{-1}\) from low to high rate sensitivity. In order to account for this behavior, we adopt the model of Marusich and Ortiz [49] which assumes a stepwise variation of the rate sensitivity exponent \(m\) while maintaining continuity of stress. Finally, we adopt a conventional power hardening law with linear thermal softening. It should be noted that, owing to the staggered integration of the coupled thermal-mechanical equations, the temperature remains fixed during a mechanical step and, therefore, plays the role of a known parameter during a stress update.

During impact, substantial amounts of heat may be generated due to the plastic working of the target and projectile materials and to friction at all the contacting surfaces. The temperatures attained can be quite high [13] and have a considerable influence on the mechanical response. The rate of heat supply due to the first plastic dissipation is estimated as

$$s = \beta \dot{W}^p,$$  \hspace{1cm} (2)$$

where \(\dot{W}^p\) is the plastic power per unit deformed volume and \(\beta\) is the fraction of plastic work converted into heat. The coefficient \(\beta\) is known to be a strong function of deformation in some materials [50]. For simplicity, however, we treat \(\beta\) as a constant. In addition, the rate at which heat is generated at frictional contacts is

$$h = -t \cdot [v].$$  \hspace{1cm} (3)$$

where \(t\) is the contact traction and \([\cdot]\) denotes the jump of a field variable. This heat must be apportioned between the bodies in contact. For simplicity, we assume that contacts are in thermal equilibrium, which implies continuity of temperatures. This condition, in conjunction with conservation of energy, supply the requisite thermal boundary conditions at frictional contacts.

The temperature distribution is governed by the energy balance equation which, upon finite element discretization, furnishes the semi-discrete system of equations [3]

$$C \dot{T} + KT = Q,$$  \hspace{1cm} (4)$$
where $T$ is the nodal temperature array, $C$ the heat capacity matrix, $K$ the conductivity matrix, and $Q$ is the heat source array. In the applications of interest here, the mechanical equations always set the critical time step for stability. It therefore suffices to lump the capacitance matrix and integrate the energy equations (4) explicitly by the forward Euler algorithm [37,38,3].

The mechanical and thermal field equations are coupled in two different ways. The mechanical response feeds into the thermal equations through the heat generation mechanisms expressed in (2) and (3). The reverse coupling comes from the softening effect of temperature on the yield stress. A staggered procedure [65,46,49] is adopted in order to account for this two-way coupling. An isothermal mechanical step is first taken based on the current distribution of temperatures, leading to an update of all mechanical variables. Since the temperatures are held constant throughout this step, they enter the constitutive relations as a parameter. The heat generated is computed from (2) and (3) and used to compute the heat source array $Q$ in (4). A rigid-conductor step is then taken at constant mechanical state leading to a new temperature distribution.

In simulations involving unconstrained plastic flow, high-order triangles provide a simple and convenient means of avoiding volumetric locking. However, high-order triangles behave poorly under severe dynamic contact conditions. As a convenient compromise, we adopt a composite six-noded triangular element [12,35]. In this element, the displacements are defined by piecewise linear interpolation over four subtriangles. In addition, the stresses and strains are assumed to vary linearly within the element. The requisite connection between displacements, strains and stresses is established by recourse to the Hu-Washizu variational principle. Owing to the choice of strain representation the element does not lock in the incompressible limit. Additionally, the piecewise linear interpolation results in an even distribution of contact reactions and lumped nodal masses, which confers the element the desired robustness under severe contact conditions.

3. Fracture and fragmentation

In contrast to approaches based on continuum damage theories, our aim is to explicitly follow the initiation and propagation of multiple cracks. These cracks can branch and coalesce and eventually lead to the formation of fragments. The creation of new surface is accomplished by allowing initially coherent element boundaries to open according to a cohesive law which models a gradual loss of strength with increasing separation. The cohesive law determines the work of separation, or fracture energy, required for the complete formation of a free surface. The cracks which result from normal impact by an axisymmetric solid can be classified into two main categories: conical and radial. Conical cracks intersect meridional planes at right angles. We refer to the cracks contained in meridional planes as radial cracks. In order to restrict the calculations to a meridional plane, thereby effecting a much desirable reduction to two spatial dimensions, we develop a hybrid formulation in which conical cracks are modelled discretely by their trace on meridional planes while radial cracks are modelled continuously. However, both types of cracks obey the same cohesive law. These aspects of the model are developed in subsequent sections.

3.1 Conical cracks

Following Camacho and Ortiz [12], we create new conical cracks, i.e., cracks whose surface is orthogonal meridional planes, by adaptively introducing cohesive elements at previously coherent element interfaces. Following the onset of fracture, element interfaces separate through an opening displacement $\delta$, a process which is resisted by cohesive tractions $t$. To further illustrate this process, consider an element interface and introduce a local orthonormal basis $(e_1, e_2)$ such that $e_1$ and $e_2$ are the unit tangent and normal vectors, respectively. Thus, in this basis $\delta_2$ is the normal opening displacement and $\delta_1$ is the sliding displacement. Correspondingly, $t_2$ is the normal traction across the cohesive interface and $t_1$ is the shear traction.

To further simplify the formulation of mixed-mode cohesive laws, we introduce an effective opening displacement

...
\[ \delta = \sqrt{\beta^2 \delta_1^2 + \delta_2^2} \equiv \sqrt{\delta^T C \delta}, \]  

(5)

where

\[ C = \begin{pmatrix} \beta^2 & 0 \\ 0 & 1 \end{pmatrix}. \]  

(6)

The parameter \( \beta \) assigns different weights to the sliding and normal opening displacements. Furthermore, we assume the existence of a free energy potential \( \phi \) for the cohesive law which depends on \( \delta \) only through the effective opening displacement \( \delta \). Under these conditions, the cohesive law reduces to

\[ t = \frac{t(\delta, q)}{\delta}, \]  

(7)

where \( q \) is a suitable set of damage parameters and

\[ t = \frac{\partial \phi}{\partial \delta} (\delta, q) \]  

(8)

is a scalar effective traction. It follows from (5) and (7) that the effective traction is

\[ t = \sqrt{\beta^2 \delta_1^2 + \delta_2^2}. \]  

(9)

This relation shows that \( \beta \) defines the ratio between the shear and the normal critical tractions. In brittle materials, this ratio may be estimated by imposing lateral confinement on specimens subjected to high-strain-rate axial compression [15,16].

Irreversibility manifests itself upon unloading. Therefore, an appropriate choice of internal variable is the maximum attained effective opening displacement \( \delta_{\text{max}} \). Loading is therefore characterized by the conditions: \( \delta = \delta_{\text{max}} \) and \( \delta \geq 0 \). Conversely, we shall say that the cohesive surface undergoes unloading when it does not undergoes loading. We assume the existence of a loading envelop defining a relation between \( t \) and \( \delta \) under conditions of loading, Fig. 1. A simple and convenient relation is furnished by the linear law

\[ t = \sigma_c \left( 1 - \frac{\delta}{\delta_c} \right), \]  

(10)

where \( \sigma_c \) is the spall strength of the material and \( \delta_c \) is a characteristic opening displacement. A standard application of the \( J \)-integral [71] gives the fracture energy \( G_c \) as the area \( \sigma_c \delta_c / 2 \) under the cohesive law. Thus, the cohesive law is fully determined by the spall strength and fracture energy of the material. We further assume unloading to the origin, giving

![Fig. 1. Two simple cohesive laws: (a) triangular envelop; (b) Smith–Ferrante envelop.](image-url)
\[ t = \frac{t_{\text{max}}}{\delta_{\text{max}}} \delta \quad \text{if} \quad \delta < \delta_{\text{max}} \quad \text{or} \quad \dot{\delta} < 0. \] (11)

The loading cohesive law and two loading-unloading paths are shown in Fig. 1.

It bears emphasis that, upon closure, the cohesive surfaces are subject to the contact unilateral constraint, including friction. We regard contact and friction as independent phenomena to be modelled outside the cohesive law. Friction may significantly increase the sliding resistance in closed cohesive surfaces. In particular, the presence of friction may result in a steady – or even increasing – frictional resistance while the normal cohesive strength simultaneously weakens.

3.2. Radial fracture

As already mentioned, in order to keep the calculations axisymmetric we model radial cracks, i.e., cracks contained within radial planes continuously. It should be emphasized that the modelling of radial cracks is essential in axisymmetric calculations. Thus, if radial cracks were not taken into account all fragments would take the form of intact rings and would therefore be unable to move out radially or rotate within meridional planes. Only by accounting for radial cracking, the fragments gradually free themselves from the axisymmetric constraint as they effect a continuous transition to a plane state of stress. In the limit of profuse radial cracking, the fragments are subject to plane stress conditions and, in particular, are capable of rotating freely within meridional planes.

We begin by assuming the particular radial crack pattern shown in Fig. 2. This pattern is characterized by a constant separation \( l \) between radial cracks, e.g., of the order of the grain size of the material. The radial cracks are acted upon by the hoop stress \( \sigma_{\theta \theta} \). Assume now that the opening of the radial cracks is governed by a cohesive law of the form described in the preceding section, Fig. 1. From the form of the cohesive law it is evident that the crack begins to open when \( \sigma_{\theta \theta} \) reaches the spall strength \( \sigma_s \) of the material. Following the attainment of this critical condition, \( \delta_s \geq 0 \) measures the opening of the cracks in the circumferential direction, and this opening is resisted by cohesive tractions. It therefore follows that \( \sigma_{\theta \theta} \) follows directly from the cohesive law and the prior history of the opening displacement \( \delta_\theta \). In particular, the hoop stress \( \sigma_{\theta \theta} \) drops to zero when the opening displacement \( \sigma_{\theta \theta} \) attains a critical value \( \delta_c \), Fig. 1, following which the state of stress remains one of plane stress for as long as the radial cracks are open, i.e., while \( \delta_\theta > 0 \). The state of stress recovers its full axisymmetric character upon crack closure, during which \( \delta_\theta = 0 \) and \( \sigma_{\theta \theta} < 0 \).

It only remains to determine \( \delta_\theta \) from the kinematics of deformation. By jointly considering the elastic-plastic deformations and radial cracking, the deformation gradients of the solid take on the multiplicative representation

\[ F = F^e F^p F^{\text{cracks}}, \] (12)

where \( F^e \) and \( F^p \) are the elastic and plastic parts of \( F \), respectively, and \( F^{\text{cracks}} \) represents the net kinematic effect of radial cracking. Since the opening of radial cracks is strictly in the circumferential direction one has

![Fig. 2. Assumed pattern of radial cracks. The cracks are uniformly spaced by a characteristic distance \( l \).](image-url)
\[ F_{\text{cracks}} = I + (\lambda_{\text{cracks}}^2 - 1)e_0 \otimes e_0, \]  

(13)

where \( \lambda_{\text{cracks}} \) is the circumferential stretching due to the opening of radial cracks and \( e_0 \) is the unit vector in the circumferential direction. After deformation, the length of a circle centered at the axis of rotational symmetry and having a radius \( R \) in its underformed configuration is

\[ 2\pi r = 2\pi R \lambda^e \lambda^p \lambda_{\text{cracks}}, \]  

(14)

where \( \lambda^e \) and \( \lambda^p \) are the elastic and plastic circumferential stretch ratios, respectively. The product \( \lambda^e \lambda^p \) measures the actual stretching of the material in the ring. In addition, since the underformed length of the fragments in the circumferential direction is assumed to be \( l \), the number \( n \) of fragments is \( 2\pi R/l \). The total length of all the deformed fragments is, therefore,

\[ L = n \lambda^e \lambda^p l = 2\pi R \lambda^e \lambda^p. \]  

(15)

Subtracting (15) from (14), and dividing by the number \( n \) of radial cracks, the average opening \( \delta_0 \) finally follows as

\[ \delta_0 = \lambda^e \lambda^p (\lambda_{\text{cracks}}^2 - 1)l. \]  

(16)

These kinematical relations evince that the process of radial cracking is necessarily coupled to the elastic-plastic deformations of the material, as expected. Thus, the elastic and plastic deformations depend on the hoop stress \( \sigma_0 \), which in turn depends through the cohesive law on the opening displacement \( \delta_0 \), which in turn depends through (16) on the elastic and plastic deformations of the material, which closes the chain of dependencies.

4. Glass rod impact experiments

Glass rod and plate impact experiments, by the sheer extent of fragmentation they typically involve, furnish a challenging validation test for the computational methodology described in the foregoing. We specifically consider the experimental configuration of Brar and Bless [8], in which a projectile strikes the end of a circular cross section glass rod. The rod has diameter of 12.7 mm, a length of 170 mm, and the projectiles are metal plates or glass rods launched by a 50 mm diameter gas gun. High speed photography of the impact revealed the presence of failure waves, defined by Brar et al. as a wave that propagates into a stressed brittle material where the material is intact ahead of the wave and comminuted behind the wave, Fig. 3. At 125 m/s impact velocity Brar et al. report a failure-wave speed of 2.3 mm/\mu s, whereas at an impact velocity of 225 mm/s the failure-wave speed rises to 3.6 mm/\mu s.

The material properties assumed in the calculations are collected in Tables 1-3. The mesh size is set at \( h = 0.5 \) mm, which results in an initial mesh of 244 nodes and 82 elements and a final mesh of 10,330 nodes and 4100 elements. We start by meshing the domain by the advancing front method, which results in fairly uniform element sizes. Simulations are conducted for an impact velocity of 210 m/s, which falls within the range of Brar et al. experiments [8].

The predicted fracture evolution is shown in Fig. 3. In the spirit of our axisymmetric model, we exhibit the meridional cracks explicitly and the radial cracks through level contours of the damage variable \( \varepsilon = \varepsilon_{\text{max}}/\varepsilon_c \). Shortly after impact, surface flaws just outside the contact region grow into the interior of the rod. Tensile waves reflected from the surface of the rod converge on its axis, which begins to burst noticeably, Fig. 3c. After the passage of some time, a roughly planar front forms which propagates down the axis of the rod and separates essentially intact material from comminuted material, Fig. 3c. This front is, therefore, the failure wave observed by Brar et al. The mean velocity of propagation of the failure wave is predicted to be 3 mm/\mu s, which is in the range of failure-wave speeds reported by Brar et al.
Fig. 3. (a) Glass rod impact experiments of Brat and Bliss [8]. Frames 1–6 are 10 μs apart. Four snapshots of the glass rod impact simulation at: (b) 0 μs; (c) 4.4 μs; (d) 9.2 μs and (e) 14.7 μs.

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<th>$v$</th>
<th>$\sigma_y$ (GPa)</th>
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Fig. 3. (a) Glass rod impact experiments of Brar and Bless [8]. Frames 1–6 are 10 μs apart. Four snapshots of the glass rod impact simulation at: (b) 0 μs; (c) 4.4 μs; (d) 9.2 μs and (e) 14.7 μs.

Table 1
Mechanical material constants

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Fracture material constants

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Table 3
Thermal constants

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5. Concluding remarks

The main focus of this communication has been to provide further illustration of the feasibility of simulating fragmentation explicitly, crack by crack. The theoretical appeal of this approach is manifold. By entrusting the nucleation, propagation, branching and other aspects of the fracture behavior of materials to a master cohesive law, the volume of phenomenology is considerably reduced relative to theories of distributed damage. In addition, cohesive theories furnish complete and bona fide representations of the fracture properties of solids. Unlike damage theories, a cohesive law introduces a well-defined fracture energy, which rids calculations of spurious mesh dependencies. In particular, finite element solutions attain proper convergence in the limit of a vanishing mesh size. This desirable property illustrates the fact that cohesive models endow materials with a characteristic length. A less widely recognized fact is that cohesive models also endow materials with an intrinsic length scale [12]. By virtue of this internal clock, the material discriminates between fast and slow loading processes, with the result that the effective response of the solid is rate dependent [63].

Another argument in favor of the explicit simulation a fragmentation is the wealth of detail which the calculations afford as regards fracture patterns. In this respect, the explicit fragmentation paradigm furnishes, perhaps for the first time, a realistic chance of computing fragment distribution sizes, crack speeds, and other features of interest from first principles of fracture mechanics. Here, we have endeavored to demonstrate the scope of the method with the aid of Brad and Bress [8] glass rod impact experiments and, as we have seen, the predicted failure wave speeds are in accord with observation. A more complete analysis of failure waves is currently in progress and will be reported in the future.

Acknowledgements

The support of the Army Research Office through contract DAAH04-96-1-0056 is gratefully acknowledged.

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