Effect of decohesion and sliding on bimaterial crack-tip fields

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Abstract. This work is concerned with the analytical characterization of the effect of bond decohesion and sliding on the fields surrounding the tip of an interface crack. We consider the two-dimensional problem of an interface crack along the bond between a pair of linearly elastic materials. The interface itself has a nonlinear constitutive property: it has maximum load-carrying capacities in both tension normal to the bond and in shear. The interface therefore has the ability to slide and separate inelastically without loss of integrity. The effects of these physically motivated assumptions are deduced and discussed. Further impetus for this study stems from the recent resurgence of interest in interfacial fracture mechanics. This interest is partly driven by the desire to understand and alleviate the pathological difficulties associated with the crack-tip fields predicted by the linear theory of elasticity. By accounting for possible interfacial nonlinear behavior, we are able to find that near-tip fields are free of the offensive properties alluded to above.

1. Introduction

In this paper, we consider the two-dimensional problem of an interface crack at the bond between a pair of linearly elastic materials. The interface itself has a constitutive property: it has maximum load-carrying capacities in both tension normal to the bond and in shear. If the interfacial yield stress in tension or shear is reached along some portion of the bond line, the half-spaces may separate and slide relative to one another over this same segment. Should the bond yield in tension, the interface’s shear yield strength may decrease, and likewise, shear yielding has the effect of reducing the bond’s ability to sustain normal stresses. More physically realistic models of interface behavior have been proposed by Needleman [1].

Problems involving interface cracks have recently been the focus of much attention. The very earliest study, carried out by Williams [2], led to the discovery of the well-known oscillatory asymptotic fields. With the exception of a few special combinations of material properties, these fields predict interpenetration of the crack faces. In the case of a pair of incompressible linearly elastic materials in plane strain, the offensive oscillations are not present. Sih and Rice [3, 4] established a connection between the singular near-tip fields and the remote conditions in particular boundary value problems.

Many investigators have sought ways of coping with the physically untenable interpenetration predicted by the linearized theory of elasticity. Comminou [5], for example, approached the analysis of interface cracks by assuming that the crack faces may be in frictionless contact over a small zone near the crack tip. Achenbach and co-workers [6] dealt with the problem by introducing a cohesive zone along the interface in front of the crack. They made a-priori assumptions on the stress distribution along the cohesive zone and demand that the stress fields surrounding the crack be free of singularities, oscillatory or
otherwise. Knowles and Sternberg [7] carried out an asymptotic analysis of an interface-crack problem within the theory of plane stress in finite elastostatics. Their study predicted stress fields near the crack tip that are singular, although free of any difficulties associated with crack face interpenetration.

Recently, Shih and Asaro [8] performed a full-field numerical analysis of a crack along a bond line between an elastic-plastic medium and a rigid substrate. They dealt with materials whose plasticity is described by a Ramberg-Osgood constitutive law, and considered the small-scale yielding problem. They were able to establish the oscillatory, nonseparable structure of the singular fields, and characterize the plastic zone shapes and sizes.

Our view of bond failure ahead of interface cracks is amenable to analytical treatment. The results reveal insight into the structure of stress fields for varying types of load conditions and yield behavior. Moreover, we investigate the implications of a crack-growth criterion based on a critical crack-tip opening displacement.

2. Formulation of the problem

In order to characterize the effect of decohesion and sliding on the fields surrounding the tip of an interface crack, we consider the two-dimensional problem of a semi-infinite crack separating two otherwise bonded dissimilar materials. Axes are chosen such that the interface lies on the line $x_2 = 0$ and the crack coincides with the ray $x_1 \leq 0$. In the interest of simplicity, we treat both materials as linearly elastic, with shear moduli $\mu^\pm$ and Poisson’s ratios $\nu^\pm$. Here and in the sequel, the labels $\pm$ refer to the constituent materials of the composite solid occupying the upper and lower half planes, respectively. Following standard practice, we introduce the constants $\kappa^\pm = 3 - 4\nu^\pm$ for plane strain, $\kappa^\pm = (3 - \nu^\pm)/(1 + \nu^\pm)$ for plane stress, and the bimaterial constant

$$\varepsilon = \frac{1}{2\pi} \log \left( \frac{\mu^- \kappa^+ + \mu^+}{\mu^+ \kappa^- + \mu^-} \right).$$

(1.1)

For physically reasonable values of the elastic constants, $(\mu^\pm > 0, 0 \leq \nu^\pm \leq 1/2) |\varepsilon| \leq (1/2\pi) \ln(3) \approx 0.175$, so that $\varepsilon$ is quite small. We will assume, without loss of generality that $\varepsilon \geq 0$.

The complex stress intensity factor adopted here is that of Shih and Asaro [8], so that the complex traction on the bond line is given by

$$\sigma_{23}(x, 0) + i\sigma_{12}(x, 0) = \frac{K_1 + iK_2}{\sqrt{2\pi x}} \left( \frac{x}{R} \right)^\kappa, \quad (x > 0),$$

(1.2)

where $R$ is some characteristic length of the problem. In particular boundary value problems with given remote loads, the ratio of $K_1$ to $K_2$ is independent of the characteristic length $R$ [8]. This feature is not found in other stress intensity factors, such as those of Erdogan [9] and Sih and Rice [4].
We will need expressions for the jump in displacement \([u]_i(x) = u_i(x, 0^+) - u_i(x, 0^-)\) along the crack faces, and these are given as

\[
[u_2](x) + i[u_1](x) = \frac{1}{\sqrt{2\pi}} \left[ \frac{\kappa^+ + 1}{2\mu^+} + \frac{\kappa^- + 1}{2\mu^-} \right] K_1 + iK_2 \frac{\sqrt{-x}}{1 + 2i} \frac{\sqrt{x}}{\cosh(\pi e)} \left( \frac{-x}{R} \right), x < 0.
\]

(1.3)

For \(\varepsilon \neq 0\), the interpenetration of the crack faces may be seen from the above expression. A necessary condition on the constants \(K_1\) and \(K_2\) that this interpenetration take place in an interval \(-r_e \leq x \leq 0\), negligibly small when compared with \(R\), is found by demanding that \([u_2](x) > 0\) for \(-R \leq x < -r_e\). A convenient and reasonable choice of the interpenetration distance is \(r_e = Re^{-\varepsilon/4e}\). \((r_e = 3.9 \times 10^{-3} R\) when \(\varepsilon = 0.1\)). No overlapping of the crack faces occurs for \(-R \leq x < -r_e\) if and only if the stress intensity factors \((K_1, K_2)\) obey

\[
K_1 + 2\varepsilon K_2 > 0, \quad \varepsilon[K_1 + K_2 + 2\varepsilon(K_2 - K_1)] > 0.
\]

(1.4)

Since \((K_1, K_2)\) depend on the loading and geometry of a particular bimaterial crack problem, (1.4) amounts to restrictions on the latter parameters. If (1.4) fails to hold, we say that the remote loading tends to close, rather than open, the crack. Note that when \(\varepsilon = 0\), (1.4) reduces to the requirement that \(K_1 > 0\), or that the mode I component of the remote loading is tensile.

The interface is assumed to possess structure conferring on it the ability to slide and separate inelastically. In deference to analytical tractability, we adopt a simple model to describe the inelasticity of the interface. We suppose that the interface is initially capable of carrying normal stresses below a maximum value \(\sigma_i\) without separation; as far as the bond’s shear strength is concerned, we assume that the bond in its virgin state can support, without sliding, shear stresses whose absolute value doesn’t exceed an amount \(\tau_e\). The initial yield levels \(\sigma_i, \tau_e\) will be altered as the bond fails. Separation on the bond-segment may cause a reduction in shear strength, and similarly, shear yielding may have the effect of lowering the maximum sustainable normal stress.

We assume that the interface fails over a region \(0 < x_i \leq R\) wherein the resolved normal and shear stresses remain at critical values \(\sigma_0\) and \(\tau_0\), with \(0 \leq \sigma_0 \leq \sigma_i\), and \(|\tau_0| \leq \tau_e\). The boundary conditions along the bond are

\[
\begin{align*}
\sigma_{12}(x, 0) &= \sigma_{22}(x, 0) = 0, \quad x < 0, \\
\sigma_{22}(x, 0) &= \sigma_0, \quad 0 < x \leq R, \\
\sigma_{12}(x, 0) &= \tau_0, \quad 0 < x \leq R, \\
[u_1](x) &= [u_2](x) = 0, \quad x > R, \\
[\sigma_{12}](x) &= [\sigma_{22}](x) = 0, \quad x > R.
\end{align*}
\]

(1.5)

\(^1\) Latin subscripts have range \((1, 2)\).
To completely define the problem, remote loading conditions need to be specified. At distances from the crack tip much greater than the size of the decohesion zone, it appears reasonable to expect that the solution for the perfectly bonded bimaterial crack be recovered. Thus, two distinct scales may be identified in this problem. The first concerns the range of dominance of the surrounding K-field, or outer singular field. This is a function of the geometry of the solid. The second provides a measure of the range of influence of the zone of decohesion. If the interface is strong enough that these two length scales are greatly dissimilar, one may regard the outer singularity, or elastic K field, as furnishing the remote conditions for the inner solution. This amounts to the requirement that

$$u_i \sim K_1^x u_i^1 + K_2^x u_i^2, \quad \sigma_{ij} \sim K_1^x \sigma_{ij}^1 + K_2^x \sigma_{ij}^2 \quad \text{as} \quad r^2 = x_1^2 + x_2^2 \to \infty,$$

(1.6)

where \((K_1^x, K_2^x)\) are the remote or applied stress intensity factors, and \((u_i^1, \sigma_{ij}^1), (u_i^2, \sigma_{ij}^2)\) are the linearly independent fields conforming to

$$\sigma_{ij}(x_1, x_2) = K_1 \sigma_{ij}^1(x_1, x_2) + K_2 \sigma_{ij}^2(x_1, x_2),$$

$$u_i(x_1, x_2) = K_1 u_i^1(x_1, x_2) + K_2 u_i^2(x_1, x_2),$$

(1.7)

if \(\sigma_{ij}\) and \(u_i\) are the stress and displacement fields for the perfectly bonded interface crack solution. (See, for example, [8].) We assume that \((K_1^x, K_2^x)\) satisfy the inequalities (1.4), so that, in the sense outlined earlier, the remote loading tends to open, rather than close the crack.

3. Analytical solution

In this section, we seek to construct solutions to the problem by superposition of perfectly bonded bimaterial cracks with tips continuously distributed over the zone of decohesion. Thus, we consider displacement and stress fields of the type

$$u_i(x_1, x_2) = \int_0^R [u_i^1(x_1 - y, x_2)k_1(y) + u_i^2(x_1 - y, x_2)k_2(y)] \, dy,$$

$$\sigma_{ij}(x_1, x_2) = \int_0^R [\sigma_{ij}^1(x_1 - y, x_2)k_1(y) + \sigma_{ij}^2(x_1 - y, x_2)k_2(y)] \, dy.$$  

(3.1)

Here, \((k_1, k_2)\) are as yet undetermined weight functions defined in the interval \((0, R)\). As mentioned previously, the functions \(u_i^1, u_i^2, \sigma_{ij}^1\) and \(\sigma_{ij}^2\) are the \(K_1\) and \(K_2\) fields for a perfectly bonded bimaterial crack. The quantities \(k_1(x) dx, k_2(x) dx\) represent the stress intensity factors of the bimaterial cracks whose tips are distributed in the interval \([x, x + dx]\). Cottrell [10] exploited a similar superposition technique in his crack-tip field studies.

Further insight into the significance of the weights \((k_1, k_2)\) may be derived from the remote load conditions. For values of \(\sqrt{x_1^2 + x_2^2} \approx R\), it follows from (3.1) that

$$u_i \sim \left[ \int_0^R k_1(y) \, dy \right] u_i^1 + \left[ \int_0^R k_2(y) \, dy \right] u_i^2,$$

$$\sigma_{ij} \sim \left[ \int_0^R k_1(y) \, dy \right] \sigma_{ij}^1 + \left[ \int_0^R k_2(y) \, dy \right] \sigma_{ij}^2.$$

(3.2)
For these expressions to be consistent with (1.6), one requires

\[ K_1^\infty = \int_0^R k_1(y) \, dy, \quad K_2^\infty = \int_0^R k_2(y) \, dy. \tag{3.3} \]

Thus, the effect of decohesion may be regarded as a redistribution of the applied stress intensity factors over the finite interval \((0, R]\).

That (3.1) identically satisfy the governing field equations is a direct consequence of the linearity of the problem. Further, it is clear that all solutions of type (3.1) conform to

\[ \sigma_{22}(x, 0) = \sigma_{12}(x, 0) = 0, \quad x < 0, \]

\[ \boxed{\left[ u_1 \right](x) = \left[ u_2 \right](x) = 0, \quad x > R, } \tag{3.4} \]

\[ \boxed{\left[ \sigma_{22} \right](x) = \left[ \sigma_{12} \right](x) = 0, \quad x > R. } \]

The stress intensity distributions \((k_1(x), k_2(x))\), along with the critical stress levels \(\sigma_0, \tau_0\) and decohesion zone-size \(R\) are determined by the remaining boundary conditions.

To this end, we begin by restricting (3.1) to the interface \((x_2 = 0)\), and obtain from (1.5)

\[ \sigma_0 + i\tau_0 = \sigma_{22}(x, 0) + i\sigma_{12}(x, 0) = \frac{R^{-i\varepsilon}}{\sqrt{2\pi}} \int_0^x \frac{k(y)}{(x - y)^{1/2 + i\varepsilon}} \, dy, \quad (0 < x \leq R), \tag{3.5} \]

in which \(k\) is the complex stress intensity distribution,

\[ k(x) = k_1(x) + ik_2(x), \quad (0 < x \leq R). \tag{3.6} \]

The characteristic length for this problem is the decohesion zone size, \(R\). Equation (3.5) furnishes a complex singular integral equation for \(k\). This equation is readily solved. (See, for example Courant and Hilbert, [11].) One has

\[ k(x) = \sqrt{\frac{2}{\pi\varepsilon}} \cosh (\pi\varepsilon) \left[ \sigma_0 + i\tau_0 \right] \left( \frac{x}{R} \right)^{-i\varepsilon}, \quad (0 < x \leq R). \tag{3.7} \]

The three unknowns, \(\sigma_0, \tau_0, \) and \(R\), remain to be determined. An appeal to the subsidiary conditions (3.3) leads to

\[ \int_0^R k(x) \, dx = \sqrt{\frac{2R}{\pi}} \cosh(\pi\varepsilon) \frac{\sigma_0 + i\tau_0}{1/2 - i\varepsilon} = K_{1\infty} + iK_{2\infty}. \tag{3.8} \]

From this equation, we find that

\[ R = \left[ \frac{\sqrt{\pi}(1/2 - i\varepsilon)(K_{1\infty} + iK_{2\infty})}{\sqrt{2} \cosh(\pi\varepsilon)(\sigma_0 + i\tau_0)} \right]^2 \tag{3.9} \]
Requiring the right-hand side of the above expression to be real-valued furnishes
\[ \alpha = \frac{\tau_0}{\sigma_0} = \frac{K_2^\infty - 2\epsilon K_1^\infty}{K_1^\infty + 2\epsilon K_2^\infty}. \] (3.10)

One can show with the aid of (1.4) that \( \alpha \geq -1 \). Equation (3.10) is used to determine \( \sigma_0 \leq \sigma_y, -\tau_y \leq \tau_0 \leq \tau_y \) according to the following rule:
\[ \begin{align*}
\sigma_0 &= \sigma_y, \quad \tau_0 = \alpha \sigma_y \text{ if } |\alpha| \leq \frac{\tau_y}{\sigma_y}, \\
\tau_0 &= \text{sign} (\alpha) \tau_y, \quad \sigma_0 = \frac{1}{|\alpha|} \tau_y \text{ if } |\alpha| > \frac{\tau_y}{\sigma_y}.
\end{align*} \] (3.11)

Owing to this relation, (3.9) may be rewritten as
\[ R = \frac{\pi (4\epsilon^2 + 1)[(K_1^y)^2 + (K_2^y)^2]}{\cosh^2(\pi \epsilon)(\epsilon^2 + 1)\sigma_y^2}, \] (3.12)
if \( \alpha \), given by (3.10), obeys \( |\alpha| \leq \tau_y/\sigma_y \), whereas
\[ R = \frac{\pi (4\epsilon^2 + 1)[(K_1^y)^2 + (K_2^y)^2]x^2}{\cosh^2(\pi \epsilon)(\epsilon^2 + 1)\tau_y^2}, \] (3.13)
in the event that \( |\alpha| > \tau_y/\sigma_y \).

4. Discussion of results

Now that the weight functions \( k_1, k_2 \), and the decohesion-zone size has been determined, the stress and displacement fields may be calculated from (3.1). Of particular interest are the stresses along the interface ahead of the zone of decohesion. One finds with the aid of (3.1), (1.2), and (3.7) that
\[ \sigma_{22}(x, 0) + i\sigma_{12}(x, 0) = \frac{\cosh (\pi \epsilon)}{\pi} \left[ \sigma_0 + i\tau_0 \right] \int_0^R \frac{1}{\sqrt{y(x-y)}} \left( \frac{x-y}{y} \right)^\kappa \, dy, \quad x \geq R. \] (4.1)

The integral appearing in the above can be expressed in terms of the incomplete beta function \( B \) as
\[ \int_0^R \frac{1}{\sqrt{y(x-y)}} \left( \frac{x-y}{y} \right)^\kappa \, dy = \int_0^{R/\epsilon} t^{-1} (1 - t)^{-\kappa} \, dt \]
\[ \equiv B_{R/\epsilon}(\lambda, 1 - \lambda), \quad (x \geq R), \quad \lambda = \frac{1}{2} - i\epsilon. \] (4.2)
NORMAL TRACTION \((K2 = 0)\)

\[
\frac{\sigma_{22}}{\sigma_0} = \frac{\cosh(\pi \varepsilon)}{\pi} (\sigma_0 + i\tau_0)B_{R/\lambda}(\lambda, 1 - \lambda), \quad (x \geq R), \quad \lambda = \frac{1}{2} - i\varepsilon.
\]

\((4.3)\)

Figure 1 shows the variation of these traction components for the case in which \(K_{2}^{\infty} = 0\); the analogous plots for \(K_{1}^{\infty} = 0\) are in Fig. 2. As may be seen, the tractions on the bond line are free of pathological oscillations.
NORMAL TRACTION ($K_1 = 0$)

\[ \sigma_{zz}/\sigma_0 \]

CRACK TIP

\[ \varepsilon = 0.1 \]

\[ \varepsilon = 0 \]

\[ x/R \]

SHEAR STRESS ($K_1 = 0$)

\[ \sigma_{12}/\tau \]

CRACK TIP

\[ \varepsilon = 0.1 \]

\[ \varepsilon = 0 \]

\[ x/R \]

*Fig. 2. Variation of tractions over bond line ($K_1^{\ast} = 0$).*

As far as crack-opening displacements are concerned, these are expressed for $x \leq R$ as

\[
\left[ u_z \right](x) + i \left[ u_t \right](x) = \frac{cosh (\pi \varepsilon)}{\pi} \left[ \frac{\kappa^+ + 1}{2\mu^+} + \frac{\kappa^- + 1}{2\mu^-} \right] \sigma_0 + i\tau_0 \int_r \left( \frac{y - x}{y} \right)^{1/2 + i\varepsilon} H(y) \, dy,
\]

(4.4)

with $H(y)$ the Heaviside step function. Crack-opening profiles for the case in which $K_1^{\ast} = 0$ are shown in Fig. 3; the case $K_1^{\ast} = 0$ is depicted in Fig. 4. We have found no evidence of
interpenetration in the region of dominance of the inner solution. Thus, the presence of decohesion and sliding in the interface appears to eliminate the troublesome aspects of the linear elasticity solution.

At $x = 0$, one has

$$\delta_2 + i\delta_1 \equiv [u_2](0) + i[u_1](0) = \frac{\cosh(\pi\varepsilon)}{\pi} \left[ \frac{\kappa^+ + 1}{2\mu^+} + \frac{\kappa^- + 1}{2\mu^-} \right] \frac{\sigma_0 + i\varepsilon_0}{1 + i2\varepsilon} R. \quad (4.5)$$
whence (3.10) gives

$$\delta_1 = \frac{\cosh (\pi \varepsilon)}{\pi(1 + 4\varepsilon^2)} \left[ \frac{\kappa^+ + 1}{2\mu^+} + \frac{\kappa^- + 1}{2\mu^-} \right] \sigma_0(\alpha - 2\varepsilon)R,$$

$$\delta_2 = \frac{\cosh (\pi \varepsilon)}{\pi(1 + 4\varepsilon^2)} \left[ \frac{\kappa^+ + 1}{2\mu^+} + \frac{\kappa^- + 1}{2\mu^-} \right] \sigma_0(1 + 2\varepsilon\alpha)R.$$

(4.6)
The latter quantity is nonnegative, provided \( \alpha \leq -1/(2\varepsilon) \) which, according to (3.10) holds if the remote load parameters \((K^\infty_1, K^\infty_2)\) obey

\[
K^\infty_1 + 2\varepsilon K^\infty_2 > 0, \quad (1 - 4\varepsilon^2)K^\infty_1 + 4\varepsilon K^\infty_2 \geq 0. \tag{4.7}
\]

Because \( \varepsilon \leq (1/2\pi) \log (3) \), the above condition is easily seen to be implied by the constraint (1.4) on \((K^\infty_1, K^\infty_2)\). Thus, (1.4) guarantees that \( [u_2](0) > 0 \).

Next, we determine the locus of points in the \((K^\infty_1, K^\infty_2)\)-plane for which a critical crack-tip opening displacement is attained, resulting in crack advance. If \( \delta_{1c}, \delta_{2c} \) represent the critical sliding and opening displacement, we assume that the crack grows if

\[
\left( \frac{\delta_1}{\delta_{1c}} \right)^n + \left( \frac{\delta_2}{\delta_{2c}} \right)^n = 1, \tag{4.8}
\]

where \( n \) is a material parameter. The desired interaction diagram is found by combining the above with (3.10), (3.11), and (4.5), (4.6). If for instance, \( n = 2 \) and \( \delta_{1c} = \delta_{2c} \equiv \delta_c \), one finds that crack advance occurs if

\[
(\tilde{K}_1^2 + \tilde{K}_2^2)(\tilde{K}_1 + 2\varepsilon \tilde{K}_2)^2 = 1, \quad \text{if } |\alpha| \leq \frac{\tau_s}{\sigma_s},
\]

\[
(\tilde{K}_1^2 + \tilde{K}_2^2)(\tilde{K}_2 - 2\varepsilon \tilde{K}_1)^2 = \left( \frac{\tau_s}{\sigma_s} \right)^2, \quad \text{if } |\alpha| \geq \frac{\tau_s}{\sigma_s}, \tag{4.9}
\]

in which \((\tilde{K}_1, \tilde{K}_2)\) are dimensionless remote-load parameters, given by

\[
\tilde{K}_j = \frac{K^\infty_j \left[ \frac{\kappa^+ + 1}{2\mu^+} + \frac{\kappa^- + 1}{2\mu^-} \right]^{1/2}}{\sqrt{\sigma_j \delta_j \cosh (\pi\varepsilon)}}, \quad (j = 1, 2). \tag{4.10}
\]
The interaction locus for the case in which \( \tau = \sigma \) is shown in Fig. 5. As may be seen, the symmetry of the curve about the diagonal \( \hat{K}_1 = \hat{K}_2 \) is lost when \( \varepsilon \neq 0 \).

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References


Résumé. On considère le problème à deux dimensions d'une fissure d'interface suivant la liaison entre deux matériaux élastiques linéaires, et la caractérisation analytique de l'effet d'une décohésion et d'un glissement de cette liaison sur le champ entourant l'extrémité de la fissure d'interface.

Ce dernier possède lui-même une propriété constitutive non linéaire et présente une capacité de chargement maximum à la fois suivant une traction normale par rapport au joint et en cisaillement. Dès lors, l'interface peut glisser ou se séparer de manière inélastique sans perte de l'intégrité de la liaison. Ces hypothèses physiques ont des effets qui sont discutés. Une telle étude est stimulée par le regain récent d'intérêt pour la mécanique de rupture des interfaces, en partie dû au désir de comprendre et d'alléger les difficultés auxquelles conduit une prédiction des champs à l'extrémité d'une fissure en se basant sur la théorie linéaire de l'élasticité. En prenant en compte la possibilité d'un comportement non linéaire de l'interface, on est à même de trouver que les champs au voisinage de l'extrémité de la fissure ne présentent pas de propriétés adverses.